Abstract Booklet

29th British Combinatorial Conference

Monday 11th – Friday 15th July 2022

Contents

Plenary Talks

Hypergraph Turán Problems in ℓ_2-Norm

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(This talk is based on joint work with Felix Christian Clemen, Bernard Lidický.)

There are various different notions measuring extremality of hypergraphs. We compare the recently introduced notion of the codegree squared extremal function with the Turán function, the minimum codegree threshold and the uniform Turán density.

The codegree squared sum $\mathrm{co}_2(G)$ of a 3-uniform hypergraph G is defined to be the sum of codegrees squared $d(x, y)^2$ over all pairs of vertices x, y. In other words, this is the square of the ℓ_2-norm of the codegree vector. We are interested in how large $\mathrm{co}_2(G)$ can be if we require G to be H-free for some 3-uniform hypergraph H. This maximum value of $\mathrm{co}_2(G)$ over all H-free n-vertex 3-uniform hypergraphs G is called the codegree squared extremal function, which we denote by $\mathrm{exco}_2(n, H)$.

We systemically study the extremal codegree squared sum of various 3-uniform hypergraphs using various proof techniques. Some of our proofs rely on the flag algebra method while others use more classical tools such as the stability method.

Monday 14:00, George Fox Lecture Theatre 1

Finite Geometry and Extremal Graph Theory

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The aim of this talk is to enlighten the "geometric" picture behind some extremal graphs: that can be fascinating itself and it can also suggest new ways to tackle the problem. One of the nicest examples are the Cayley graphs $\Gamma(G, S)$, when G is the additive group of a vector space over a finite field. In this way, we can look from another prospective some remarkable properties, such as being pseudorandom or clique-free, providing a different proof of known results and suggesting new ways to tackle long standing open problems. Some new constructions will be presented.

Tuesday 09:00, George Fox Lecture Theatre 1

Convex and Combinatorial Tropical Geometry

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(This talk is based on joint work with Grigoriy Blekherman, Felipe Rincón, Rainer Sinn, and Cynthia Vinzant.)

Tropical geometry is the geometry over the max-plus algebra, and it is a degeneration or limit of classical geometric objects under the logarithm or valuation map. We will discuss how to tropicalize algebraic sets, semialgebraic sets, and convex sets, and highlight an application to the truncated moment problem in real algebraic geometry.

Explicit bounds in graph minors

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(This talk is based on joint work with Ken-ichi Kawarabayashi and Robin Thomas.)

Robertson and Seymour proved a theorem approximately characterizing all graphs excluding some fixed graph H as a minor, a result which has had an enormous impact on the field with numerous applications in graph theory and theoretical computer science. The proof is notable for its complexity, stretching over a series of 16 papers. Moreover, the proof does not give explicit bounds on the parameters involved.

We present recent work yielding new and simplified proofs for the main results in graph minors series. Beyond simplifying the results, we also for the first time give explicit bounds on the parameters involved.

Wednesday 09:00, George Fox Lecture Theatre 1

Fair partitions

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A substantial number of results and conjectures deal with the existence of a set of prescribed type which contains a fair share from each member of a finite collection of objects in a space, or the existence of partitions in which this is the case for every part. Examples include the Ham-Sandwich Theorem in Measure Theory, the Hobby-Rice Theorem in Approximation Theory, the Necklace Theorem and the Ryser Conjecture in Discrete Mathematics, and more. The techniques in the study of these results combine combinatorial, topological, geometric, probabilistic and algebraic tools. I will describe the topic, focusing on several recent existence results and their algorithmic aspects.

Thursday 09:00, George Fox Lecture Theatre 1

High Dimensional Expanders in Theoretical Computer Science

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Weizmann Institute of Science

Expander graphs have been studied in many areas of mathematics and in computer science with versatile applications, including coding theory, networking, computational complexity and geometry.

High-dimensional expanders are a generalization that has been studied in recent years and their promise is beginning to bear fruit. In the talk, I will survey some powerful local to global properties of high-dimensional expanders, and describe several interesting applications, ranging from convergence of random walks to construction of locally testable codes that prove the c^3 conjecture (namely, codes with **c**onstant rate, **c**onstant distance, and **c**onstant locality).

Thursday 13:45, George Fox Lecture Theatre 1

Rainbow subgraphs and their applications

Alexey Pokrovskiy

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(This talk is based on joint work with Alp Müyesser.)

A rainbow subgraph in an edge-coloured graph is one in which all edges have different colours. This talk will be about finding rainbow subgraphs in colourings of graphs that come from groups. An old question of this type was asked by Hall and Paige. Their question was equivalent to the following "Let G be a group of order n and consider an edge-coloured $K_{n,n}$, whose parts are each a copy of G and with the edge $\{x, y\}$ coloured by the group element xy. For which groups G, does this coloured $K_{n,n}$ contain a perfect rainbow matching?" This question is equivalent to asking "which groups G contain a complete mapping" and also "which multiplication tables of groups contain transversals". Hall and Paige conjectured that the answer is " all groups in which $\prod_{x \in G} x \in G'$" (where G' is the commutator subgroup of the group). They proved that this is a necessary condition, so the main part of the conjecture is to prove that "$\prod_{x \in G} x \in G' \implies$ the corresponding $K_{n,n}$ has a perfect rainbow matching". The Hall-Paige Conjecture was confirmed in 2009 by Wilcox, Evans, and Bray with a proof using the classification of finite simple groups. Recently, Eberhard, Manners, and Mrazović found an alternative proof of the conjecture for sufficiently large groups using ideas from analytic number theory. Their proof gives a very precise estimate on the number of complete mappings that each group has.

In this talk, a third proof of the conjecture will be presented using a different set of techniques, this time coming from probabilistic combinatorics. This proof only works for sufficiently large groups, but generalizes the conjecture in a new direction. Specifically we not only characterize when the edge coloured $K_{n,n}$ contains a perfect rainbow matching, but also when random subgraphs of it contain a perfect rainbow matching.

This extension has a number of applications, such as to problems of Snevily, Cichacz, Tannenbaum, Evans, and Patrias-Pechenik.

Friday 09:00, George Fox Lecture Theatre 1

Linear programming and the circuit imbalance measure

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(This talk is based on joint work with Daniel Dadush, Sophie Huiberts, Cedric Koh, and Bento Natura.)

The existence of a strongly polynomial algorithm for linear programming (LP) is a fundamental open question in optimization. Given an LP in the standard equality form

$$\langle c, x\rangle \text{ s.t. } Ax = b\,,\ x \geq 0\,,$$

for $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, such an algorithm would perform poly(n, m) arithmetic operations. Strongly polynomial algorithms are known for a range of network optimization problems. Two significant steps towards general LP are Tardos's poly$(n, m, \log \Delta_A)$ algorithm from 1986 and a poly$(n, m, \log \bar{\chi}_A)$ interior point method by Vavasis and Ye from 1996. Here, Δ_A is the maximum subdeterminant of the integer constraint matrix, and $\bar{\chi}_A$ is a geometric condition number associated with the matrix A.

We give an overview of recent developments that strengthen and extend these results. A key object of our study is the *circuit imbalance measure* κ_A that bounds the ratios of the entries of support-minimal vectors in the kernel of A. We exhibit combinatorial properties and proximity results of linear programs that can be used to design new exact LP algorithms. In particular, we present new circuit augmentation algorithms, and derive improved bounds on the circuit diameter of polyhedra.

Friday 15:10, George Fox Lecture Theatre 1

Intersection problems in Extremal Combinatorics: theorems, techniques and questions old and new

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The study of intersection problems in Extremal Combinatorics dates back perhaps to 1938, when Paul Erdős, Chao Ko and Richard Rado proved the (first) 'Erdős-Ko-Rado theorem' on the maximum possible size of an intersecting family of k-element subsets of a finite set. Since then, a plethora of results of a similar flavour have been proved, for a range of different mathematical structures, using a wide variety of different methods. Structures studied in this context have included families of vector subspaces, families of graphs, subsets of finite groups with given group actions, and of course uniform hypergraphs with stronger or weaker intersection conditions imposed. The methods used have included purely combinatorial methods such as shifting/compressions, algebraic methods (including linear-algebraic, Fourier analytic and representation-theoretic), and more recently, analytic, probabilistic and regularity-type methods. As well as being natural problems in their own right, intersection problems have connections with many other parts of Combinatorics and with Theoretical Computer Science (and indeed with many other parts of Mathematics), both through the results themselves, and the methods used. We will survey a selection of results, methods and open problems in this area.

Minisymposium: Extremal Combinatorics

Extremal problems in hypergraphs with quasirandom links

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(This talk is based on joint work with S. Berger, S. Piga, Chr. Reiher, and V. Rödl.)

Extremal problems for 3-uniform hypergraphs concern the maximum cardinality of a set E of 3-element subsets of a given n-element set V such that for any ℓ elements of V at least one triple is missing in E. This innocent looking problem is still open, despite a great deal of effort over the last 80 years. We consider a variant of the problem by imposing additional restrictions on the distribution of the 3-element subsets in E, which are motivated by the theory of *quasirandom hypergraphs*. These additional assumptions yield a finer control over the corresponding extremal problem. In fact, this leads to many interesting and more manageable subproblems, some of which were already considered by Erdős and Sós in the 1980ies. In this talk we consider hypergraphs whose vertices have quasirandom link graphs and report on recent progress for the corresponding extremal problems.

Tuesday 15:50, George Fox Lecture Theatre 1

The n-queens problem

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(This talk is based on joint work with Peter Keevash.)

The n-queens problem asks how many ways there are to place n queens on an $n \times n$ chessboard so that no two queens can attack one another, and the toroidal n-queens problem asks the same question where the board is considered on the surface of a torus. Let $Q(n)$ denote the number of n-queens configurations on the classical board and $T(n)$ the number of toroidal n-queens configurations. The toroidal problem was first studied in 1918 by Pólya who showed that $T(n) > 0$ if and only if $n \equiv 1, 5 \mod 6$. Much more recently Luria showed that $T(n) \leq ((1 + o(1))ne^{-3})^n$ and conjectured equality when $n \equiv 1, 5 \mod 6$. We prove this conjecture, prior to which no non-trivial lower bounds were known to hold for all (sufficiently large) $n \equiv 1, 5 \mod 6$. We also show that $Q(n) \geq ((1 + o(1))ne^{-3})^n$ for all $n \in \mathbb{N}$ which was independently proved by Luria and Simkin and, combined with our toroidal result, completely settles a conjecture of Rivin, Vardi and Zimmerman regarding both $Q(n)$ and $T(n)$.

In this talk we'll discuss some of the methods used to prove these results. A crucial element of this is translating the problem to one of counting matchings in a 4-partite 4-uniform hypergraph. Our strategy combines a random greedy algorithm to count 'almost' configurations with a complex absorbing strategy that uses ideas from the methods of randomised algebraic construction and iterative absorption.

Tuesday 16:25, George Fox Lecture Theatre 1

Extremal product free sets in groups

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(This talk is based on joint work with Peter Keevash and Dor Minzer.)

Let G be a finite groups. A subset $A \subseteq G$ is said to be *product free* if for each two elements $a, b \in A$ their product ab is not in A. In this talk we improve upon works of Gowers and Eberhard by determining the largest possible size of a product free subset of the alternating group A_n.

While this problem is group theoretic in nature, its solution resembles the theory of Erdős–Ko–Rado type problems. Indeed, in both cases the solutions can be described as dictators.

Our proof involves two main techniques:

1. A dichotomy between dictatorial structure and a pseudorandomness notion known as globalness.
2. A recent powerful tool called *Hypercsontractivity for global functions*, which allows going beyond spectral gap when studying pseudorandomness and expansion properties of graphs.

Tuesday 17:00, George Fox Lecture Theatre 1

On the Ryser-Brualdi-Stein conjecture

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The Ryser-Brualdi-Stein conjecture states that every Latin square of order n should have a partial transversal with $n-1$ elements, and a full transversal if n is odd. In 2020, Keevash, Pokrovskiy, Sudakov and Yepremyan improved the long-standing best known bounds on this conjecture by showing that a partial transversal with $n - O(\log n / \log \log n)$ elements always exists. In this talk, I will discuss how to show, for large n, that a partial transversal with $n-1$ elements always exists.

Minisymposium: Matroids and Combinatorial Geometry

Tuesday 15:15, George Fox Lecture Theatre 2

Maximum likelihood thresholds via graph rigidity

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(This talk is based on joint work with Sean Dewar, Steven Gortler, Tony Nixon, Meera Sitharam, and Louis Theran.)

The maximum likelihood threshold of a graph is the minimum number of samples required to guarantee almost sure existence of the maximum likelihood estimate in the corresponding graphical model. In this talk, I will discuss a rigidity-theoretic interpretation of this problem and show how it leads to some classification results.

Algorithms for counting realisations of minimally rigid graphs

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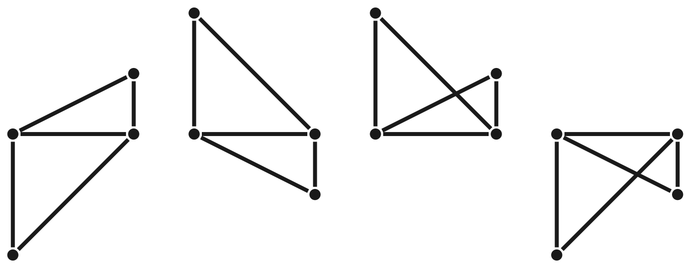
(This talk is based on joint work with Jose Capco, Matteo Gallet, Boulos El Hilany, Christoph Koutschan, Niels Lubbes, Josef Schicho.)

Minimally rigid graphs allow only finitely many non-congruent realisations for a given generic choice of edge lengths. For instance the minimally rigid graph on four vertices has four realisations in the plane up to rotations and translations when edge lengths are chosen generically (see figure).



In recent years we have investigated algorithms for counting the number of such realisations in the plane and on the sphere. While the algorithms are purely combinatorial the proofs are based on algebraic geometry. In this talk we give an overview on those algorithms and we point out their differences and what they have in common. Furthermore, we present computational results.

As a main part we report on recent progress on improving the algorithms using some combinatorial properties. In particular we show how graph splittings can be used for speeding up recursive computations. When a minimally rigid graph allows a suitable splitting we can use trajectories of motions of flexible graphs to determine the number of realisations by reducing the problem to smaller graphs.

Tuesday 16:25, George Fox Lecture Theatre 2

Some geometry of delta-matroids

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(This talk is based on joint work with Chris Eur, Matt Larson and Hunter Spink.)

Some of my favourite ways to see matroids are as matroid basis polytopes, i.e. the convex hull of basis indicator vectors, and as Bergman fans, cone complexes dual to parts of these polytopes. These viewpoints are fruitful because of connections not only to optimisation but also to algebraic geometry. It's now becoming clear how to use the same techniques for delta-matroids. I'll describe, based on joint work in progress, a couple ways these geometric connections lead to combinatorial consequences.

Tuesday 17:00, George Fox Lecture Theatre 2

Global rigidity of triangulated manifolds

Shin-ichi Tanigawa

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(This talk is based on joint work with James Cruickshank and Bill Jackson.)

The rigidity of triangulated surfaces is a classical topic in discrete geometry. In this work, we prove that if G is the graph of a connected triangulated $(d-1)$-manifold, for $d \geq 3$, then G is generically globally rigid in $\mathbb{R}^d$ if and only if it is $(d+1)$-connected and, if $d = 3$, G is not planar. The special case $d = 3$ resolves a conjecture of Connelly.

Minisymposium: Designs and Algebraic Structures

Switching for 2-designs

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(This talk is based on joint work with Dean Crnković.)

In this talk, we introduce a switching for 2-designs described in [1]. We illustrate the method by applying it to some symmetric $(64, 28, 12)$ designs. Further, we show that this type of switching can be applied to any symmetric design related to a Bush-type Hadamard matrix. We apply the switching to the designs constructed in [2, 3, 4] and construct symmetric $(36, 15, 6)$ designs leading to new Bush-type Hadamard matrices of order 36, and symmetric $(100, 45, 20)$ designs yielding Bush-type Hadamard matrices of order 100. We show that switching introduced in this talk can be applied directly to orbit matrices.

[1] D. Crnković, A. Švob, Switching for 2-designs, Des. Codes Cryptogr., to appear.

[2] Z. Janko, The existence of a Bush-type Hadamard matrix of order 36 and two new infinite classes of symmetric designs, J. Combin. Theory Ser. A 95 (2001), 360–364.

[3] Z. Janko, H. Kharaghani, A block negacyclic Bush-type Hadamard matrix and two strongly regular graphs, J. Combin. Theory Ser. A 98 (2002), 118–126.

[4] Z. Janko, H. Kharaghani, V. D. Tonchev, Bush-type Hadamard matrices and symmetric designs, J. Combin. Des. 9 (2001), 72–78.

Pairwise balanced designs and periodic Golay pairs

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(This talk is based on joint work with Doris Dumičić Danilović, Ronan Egan and Andrea Švob.)

In this talk we exploit a relationship between certain pairwise balanced designs (PBDs) with v points and periodic Golay pairs (PGPs) of length v, to classify PGPs of length less than 40 (see [1]). PBDs are constructed using orbit matrices of subgroups of a cyclic group acting on the designs, which corresponds to some compression techniques which apply to complementary sequences (see [2]). We use similar tools to construct new PGPs of lengths greater than 40 where classifications remain incomplete, and demonstrate that under some extra conditions on an automorphism group of the corresponding PBD, a PGP of length 90 will not exist. Length 90 remains the smallest length for which existence of a periodic Golay pair is undecided. Further, we show that under certain conditions the incidence and orbit matrices of PBDs related to PGPs span quasi-cyclic self-orthogonal codes.

References

[1] D. Crnković, D. Dumičić Danilović, R. Egan, A. Švob, Periodic Golay pairs and pairwise balanced designs, J. Algebraic Combin. 55 (2022), 245-257.

[2] D. Ž. Đoković, I. S. Kotsireas, Compression of periodic complementary sequences and applications, Des. Codes Cryptogr. 74 (2015), 365–377.

Tuesday 16:25, George Fox Lecture Theatre 5

Finding geometries in the power graphs of simple groups

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The *power graph* of a finite group G has vertex set G, with two elements joined by an edge if one is a power of the other. It has the defects (from some points of view) that the identity is joined to all other vertices, and there are many pairs of *twin vertices* (with the same neighbourhood except possibly one another). So it is natural to remove the identity and small components, and shrink the twin classes recursively until no pairs of twins remain.

It may happen that this reduces the graph to a single vertex. It is known for which simple groups this happens (modulo some probably very difficult number-theoretic problems); these are certain groups $\mathrm{PSL}(2, q)$ and $\mathrm{Sz}(q)$ together with $\mathrm{PSL}(3, 4)$. For other groups the result may be an interesting graph. For the Mathieu group M_{11}, for example, we find lurking within it the incidence graph of a partial linear space with 165 points, three points on each line and four lines through each point, whose incidence graph has diameter and girth equal to 10.

I will report on similar investigations of other simple groups. This is still in the exploratory stage.

Tuesday 17:00, George Fox Lecture Theatre 5

Resistance distance in the context of association schemes and coherent configurations

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(This talk is based on joint work with Peter Cameron and Michael Kagan.)

Let Ω be a finite set. Any graph whose vertex-set is Ω defines a partition of $\Omega \times \Omega$ into three parts: the diagonal, the edges and the non-edges. The corresponding adjacency matrices are I, A and $J - A - I$, where I is the identity matrix and J is the all-1 matrix. If the set of real linear combinations of these is closed under multiplication then the graph is *strongly regular*.

This idea can be generalized to partitions of Ω with more parts. Usually we insist that the diagonal is a union of parts, and that if A is the adjacency matrix of any part then its transpose $A^\top$ is also the adjacency matrix of a part. Closure under multiplication gives a *coherent configuration*. A coherent configuration in which all adjacency matrices are symmetric is an *association scheme*. Symmetry and closure under Jordan multiplication gives a *Jordan scheme*: see [5].

The set of partitions of $\Omega \times \Omega$ is partially ordered by refinement. The Weisfeiler–Leman algorithm was introduced in [7] to find the coarsest coherent configuration which refines a given graph. If the graph has large diameter, this algorithm generally takes many steps to stabilize.

Following some work of Biggs [2], Kagan had an idea for an algorithm that would take fewer steps: see [4]. This uses the idea of *resistance distance* in a graph, which is a metric which has been shown to be more useful than graph distance in the context of optimal incomplete-block designs: see [1, 6].

Kagan and Klin showed in [3] that the proposed *resistance-distance transform* (RDT) reduces many distance-regular graphs to the corresponding association scheme in a single step. Cameron proved that this is true for any graph for which the powers of A define an association scheme.

Unfortunately, the original definition of RDT does not always refine the original partition if the graph is not distance-regular. Furthermore, the refinements for a graph and its complement may be different. To overcome these difficulties, our proposed RDT2 starts with variables, one on each edge and another on each non-edge. Using these as conductances, resistance distances are then calculated as rational functions of the variables.

[1] R. A. Bailey and Peter J. Cameron: Combinatorics of optimal designs. In: *Surveys in Combinatorics 2009* (eds. S. Huczynska, J. D. Mitchell and C. M. Roney-Dougal). London Mathematical Society Lecture Notes Series, **365**, Cambridge University Press, Cambridge, 2009, pp. 19–73.

[2] Norman L. Biggs: Potential theory on distance-regular graphs. *Combinatorics, Probability and Computing*, **2** (1993), 243–255.

[3] Mikhail Kagan and Misha Klin: Resistance-distance transform (RDT) in the context of Weisfeiler–Leman stabilization (WLS). Talk presented at the conference on 'Regularity and Symmetry' in Pilsen in 2018.

[4] Mikhail Kagan and Brian Mata: A physics perspective on resistance distance for graphs. *Mathematics in Computer Science*, **13** (2019), 103–115.

[5] B. V. Shah: A generalisation of partially balanced incomplete-block designs. *Annals of Mathematical Statistics*, **30** (1959), 1041–1050.

[6] T. Tjur: Block designs and electrical networks. *Annals of Statistics*, **19** (1991), 1010–1027.

[7] B. Yu. Weisfeiler and A. A. Leman: Reduction of a graph to a canonical form and an algebra which appears in the process. *Scientific-Techincal Investigations, Series 2*, **9** (1968), 12–16.

Minisymposium: Probabilistic Combinatorics

The k-th shortest path in an edge-weighted K_n

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(This talk is based on joint work with Stephanie Gerke.)

Suppose we weight the edges of the complete graph K_n with independent exponential Exp(1) random weights, pick two distinct vertices s and t, and then successively construct, and then remove, the edges of minimal weight s-t paths. We describe asymptotically the distributions of the weights of the first k paths obtained in this process. In particular we show that the mean weight of the kth path is

$$\frac{1}{n}\big(\log n + \gamma + 2\zeta(3) + 2\zeta(5) + \cdots + 2\zeta(2k-1) + o(1)\big)$$

as $n \to \infty$ when k is a constant, and where γ is the Euler–Mascheroni constant and $\zeta(s)$ is the Riemann zeta function.

Hypergraph matchings with(out) conflicts

Stefan Glock

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(This talk is based on joint work with Felix Joos, Jaehoon Kim, Marcus Kühn and Lyuben Lichev.)

A celebrated theorem of Pippenger, and Frankl and Rödl states that every almost-regular, uniform hypergraph $\mathcal{H}$ with small maximum codegree has an almost-perfect matching. We extend this result by obtaining a "conflict-free" matching, where conflicts are encoded via a collection $\mathcal{C}$ of subsets $C \subseteq E(\mathcal{H})$. We say that a matching $\mathcal{M} \subseteq E(\mathcal{H})$ is conflict-free if $\mathcal{M}$ does not contain an element of $\mathcal{C}$ as a subset. Under natural assumptions on $\mathcal{C}$, we prove that $\mathcal{H}$ has a conflict-free, almost-perfect matching. This has many applications, one of which yields new asymptotic results for so-called "high-girth" Steiner systems. Our main tool is a random greedy algorithm which we call the "conflict-free matching process". Similar results have been proved independently by Delcourt and Postle.

Erdős-Renyi shotgun reconstruction

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(This talk is based on joint work with Tom Johnston, Alexander Roberts, and Alex Scott.)

We say that a graph G is reconstructible from its r-neighbourhoods if all graphs H having the same collection of r-balls as G up to isomorphism, are isomorphic to G. We are interested in the reconstruction of the Erdos-Renyi graph $G(n,p)$ for a wide range of values of r, aiming to determine the values of p for which $G(n,p)$ is r-reconstructible with high probability. Mossel and Ross [3] considered this problem in the sparse case where $p = C/n$, and they also considered reconstruction in the dense case where $p \gg 1/n$, and showed that the the graph $G(n,p)$ can be reconstructed from its 3-neighbourhoods with high probability provided that $p \gg \log^2(n)/n$. Later, Gaudio and Mossel [1] studied reconstruction from the 1- and 2-neighbourhoods, giving bounds on the values of p for which $G(n,p)$ is reconsructible. For 1-neighbourhoods, this was improved very recently by Huang and Tikhomirov [2] who determined the correct threshold up to logarithmic factors, around $n^{-1/2}$.

In this talk we will show new bounds on p for the r-reconstructibility problem in $G(n,p)$. We improve the bounds for 2-neighbourhoods given by Gaudio and Mossel by polynomial factors. We also improve the result of Huang and Tikhomirov for 1-neighbourhoods, showing that the logarithmic factor is necessary. Finally, we determine the exact thresholds for r-reconstructibility for $r \geq 3$.

[1] Julia Gaudio, and Elchanan Mossel. "Shotgun assembly of Erdős-Rényi random graphs." Electronic Communications in Probability 27 (2022): 1–14.

[2] Han Huang, and Konstantin Tikhomirov. "Shotgun assembly of unlabeled erdos-renyi graphs." arXiv preprint arXiv:2108.09636 (2021).

[3] Elchanan Mossel, and Nathan Ross. "Shotgun assembly of labeled graphs." IEEE Transactions on Network Science and Engineering 6, no. 2 (2017): 145–157.

Thursday 17:00, George Fox Lecture Theatre 1

The Ising model on line graphs

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(This talk is based on joint work with Martin Dyer, Marc Heinrich and Haiko Müller (Leeds).)

Line graphs are well studied in graph theory. They also occasionally appear as crystal lattices arising in nature, such as the the kagome and pyrochlore lattices. It seems natural, then, to study models from statistical physics in the context of line graphs. The Ising model is the most intensively studied such model. It is defined on a base graph G. Configurations of the model are assignments $V(G) \to \{-1, +1\}$ of 'spins' to the vertices of G. In the antiferromagnetic case, adjacent spins prefer to differ, so configurations with a large number of edges of disagreement are assigned higher weight, and have correspondingly higher probability of occurrence in the 'Gibbs distribution' on configurations. There is a parameter called 'temperature' that controls the strength of interaction along edges; the lower the temperature the stronger the interactions. At absolute zero, only the configurations of highest weight occur; these are the 'ground states', which in our case are the maximum cuts in G.

I'll start with a result obtained jointly with Martin Dyer, Marc Heinrich and Haiko Müller concerning the antiferromagnetic Ising model on line graphs. Specifically, we studied the mixing time (time to near-stationarity) of a certain simple 'Glauber' dynamics on the configurations, which changes just one vertex spin in each time step. The informal statement is that Glauber dynamics mixes in polynomial time (in the number of vertices in G) at any non-zero temperature. The main tool used in establishing this result is the canonical paths method, specifically the 'winding' technology of McQuillan. I'll describe subsequent work by others in this direction, based on more recent techniques such as interpolation along lines in zero-free regions of the partition function, and spectral independence. This work is more analytical and less combinatorial in flavour, so these improvements will not receive the attention due to them. The phenomenon of mixing at all temperatures is intriguing and seems worth studying further.

Minisymposium:
Additive Combinatorics

Thursday 15:15, George Fox Lecture Theatre 2

Subsets of $\mathbb{F}_p^n \times \mathbb{F}_p^n$ without L-shaped configurations

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I will discuss the difficult problem of proving reasonable bounds in the multidimensional generalization of Szemerédi's theorem and describe a proof of such bounds for sets lacking nontrivial configurations of the form $(x, y), (x, y + z), (x, y + 2z), (x + z, y)$ in the finite field model setting.

Thursday 15:50, George Fox Lecture Theatre 2

Quasirandomness for latin squares and counting transversals

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(This talk is based on joint work with Sean Eberhard and Rudi Mrazović.)

A *latin square* is an $n \times n$ grid filled with symbols $\{1, \ldots, n\}$ such that every symbol appears once in every row and in every column. A *transversal* of a latin square is a selection of n grid cells, comprising one from each row, one from each column, and one of each symbol. An old conjecture of Ryser asserts that every latin square of odd order has a transversal.

A recent result of Kwan shows that a randomly chosen latin square has a transversal, almost surely. I will discuss an analogue of this result (with a completely different proof) for latin squares which are "quasirandom" in a certain sense, meaning roughly that they do not resemble the multiplication table of any abelian group.

In this case we are even able to count the number of transversals, asympotically, using techniques resembling the circle method from analytic number theory.

The Typical Structure of Sets with Small Sumset

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(This talk is based on joint work with Marcelo Campos, Mauricio Collares, Rob Morris and Victor Souza.)

One of the central objects of interest in additive combinatorics is the sumset $A + B = \{a + b : a \in A, b \in B\}$ of two sets A, B of integers. Our main theorem, which improves results of Green and Morris, and of Mazur, implies that the following holds for every fixed $\lambda > 2$ and every $k > (\log n)^4$: if ω goes to infinity as n goes to infinity (arbitrarily slowly), then almost all sets $A \subseteq [n]$ with $|A| = k$ and $|A + A| < \lambda k$ are contained in an arithmetic progression of length $\lambda k/2 + \omega$.

Finding large additive and multiplicative Sidon sets in sets of integers

Akshat Mudgal

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(This talk is based on joint work with Yifan Jing.)

Given natural numbers s and k, we say that a finite set X of integers is an additive $B_s[k]$ set if for any integer n, the number of solutions to the equation

$$n = x_1 + ... + x_s,$$

with $x_1, ..., x_s$ lying in X, is at most k, where we consider two such solutions to be the same if they differ only in the ordering of the summands. We define a multiplicative $B_s[k]$ set analogously. These sets have been studied thoroughly from various different perspectives in combinatorial and additive number theory. For instance, even in the case $s = 2$ and $k = 1$, wherein such sets are referred to as Sidon sets, the problem of characterising the largest additive $B_s[k]$ set in $\{1, 2, ..., N\}$ remains a major open question in the area.

In this talk, we consider this problem from an arithmetic combinatorial perspective, and so, we show that for every natural number s and for every finite set A of integers, the largest additive $B_s[1]$ subset B of A and the largest multiplicative $B_s[1]$ subset C of A satisfy

$$\max\{|B|, |C|\} \gg_s |A|^{\eta_s/s},$$

where $\eta_s \gg (\log\log s)^{1/2-o(1)}$.

Minisymposium:
Induced Subgraphs

Induced subgraphs and tree decompositions

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(This talk is based on joint work with Maria Chudnovsky, Sepehr Hajebi, and Sophie Spirkl.)

A *tree decomposition* of a graph G is a tree T together with a map $\chi : V(T) \to 2^{V(G)}$ that roughly organizes the vertices of G into a "tree-like" structure. The *treewidth* of G is a graph parameter that uses tree decompositions to measure how "close to a tree" G is. Graphs with small treewidth have nice structural and algorithmic properties; for example, many NP-hard algorithmic problems can be solved in polynomial time in graphs with constant or logarithmic treewidth. As part of the Graph Minors Project, Robertson and Seymour proved a complete characterization of graphs with constant treewidth for graph classes defined by forbidden minors. In contrast, although graph classes defined by forbidden induced subgraphs (called *hereditary graph classes*) are the subject of much interest and research in structural graph theory, not much is currently understood regarding which hereditary graph classes have bounded treewidth. Recently, Korhonen provided a complete characterization of constant treewidth in the case of bounded maximum degree. In this talk, we discuss recent results proving that certain hereditary graph classes with unbounded degree have constant or logarithmic treewidth. These results each rely on iteratively decomposing graphs along sequences of well-chosen "non-crossing" cutsets, along with other structural tools and properties.

Thursday 15:50, George Fox Lecture Theatre 5

An algorithmic weakening of the Erdős-Hajnal conjecture

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(This talk is based on joint work with Stéphan Thomassé, Xuan Thang Tran, and Rémi Watrigant.)

We explore the approximability of the Maximum Independent Set problem in graphs excluding a fixed H as an induced subgraph (henceforth, H-free graphs). We propose the *improved approximation conjecture*: For every graph H, there is a constant $\delta > 0$ such that Maximum Independent Set can be $n^{1-\delta}$-approximated in H-free n-vertex graphs, in randomized polynomial time. Such an approximation algorithm in general graphs would imply the unlikely complexity-theoretic collapse RP=NP. The improved approximation conjecture is weaker than an effective version of the Erdős-Hajnal conjecture, where a large enough independent set or clique shall be output in polynomal time. Like for the Erdős-Hajnal conjecture, the set of graphs H for which the improved approximation conjecture is established is closed under substitution. In an attempt to match the known approximation ratio of the triangle-free case with an algorithmic barrier, we present a strong inapproximability result making use of triangle-free constructions with small independence number.

Short induced cycles in planar graphs

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In 1975, Pippenger and Golumbic instigated the study of the maximum number of induced copies of a small graph H which can be contained in an n-vertex graph G. This problem has received considerable attention over the years, yet there remain many small graphs H for which the maximum is not known even asymptotically, including the path on four vertices and many graphs on five vertices. The case where H is a cycle is of particular interest and was solved for 5-cycles for large enough n by Balogh, Hu, Lidický, and Pfender in 2016.

In this talk we will discuss recent progress on the analogous problem in the setting where the graphs G and H are planar. We will focus on the cases where H is the 4-, 5-, or 6-cycle, in each case giving exactly, for sufficiently large n, the maximum number of induced copies of H that can be contained in a planar graph on n vertices, and classifying the graphs which achieve this maximum.

Understanding graphs with no long claws

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A classic result of Alekseev asserts that for connected H the Max Independent Set (MIS) problem in H-free graphs in NP-hard unless H is a path or a subdivided claw. Recently we have witnessed some great progress in understanding the complexity of MIS in P_t-free graphs. The situation for forbidden subdivided claws is, however, much less understood.

During the talk we will present some recent advances in understanding the structure of graphs with no long induced claws, and their applications in desgining algorithms for MIS and related problems [1, 2, 3].

[1] Tara Abrishami, Maria Chudnovsky, Cemil Dibek, and Paweł Rzążewski. Polynomial-time algorithm for maximum independent set in bounded-degree graphs with no long induced claws. In Niv Buchbinder Joseph (Seffi) Naor, editor, *Proceedings of the 2022 ACM-SIAM Symposium on Discrete Algorithms, SODA 2022, Virtual Conference, January 9-12, 2022*, pages 1448–1470. SIAM, 2022. https://doi.org/10.1137/1.9781611977073.61

[2] Maria Chudnovsky, Marcin Pilipczuk, Michał Pilipczuk, and Stéphan Thomassé. Quasi-polynomial time approximation schemes for the Maximum Weight Independent Set Problem in H-free graphs. In Shuchi Chawla, editor, *Proceedings of the 2020 ACM-SIAM Symposium on Discrete Algorithms, SODA 2020, Salt Lake City, UT, USA, January 5-8, 2020*, pages 2260–2278. SIAM, 2020. https://doi.org/10.1137/1.9781611975994.139

[3] Konrad Majewski, Tomáš Masařík, Jana Novotná, Karolina Okrasa, Marcin Pilipczuk, Paweł Rzążewski, Marek Sokołowski. Max Weight Independent Set in graphs with no long claws: An analog of the Gyárfás' path argument arXiv:2203.04836, 2022. https://arxiv.org/abs/2203.04836

Contributed Talks

Partial Multi-colourings

Jan van den Heuvel

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London School of Economics and Political Science

(This talk is based on joint work with Xinyi Xu.)

Suppose you have a graph G for which the vertices can be properly coloured with t colours, but you only have $s < t$ colours available. Then it is an easy observation that you can still properly colour at least a fraction $\frac{s}{t}$ of the vertices of G. (More formally: there exists an induced subgraph H of G such that H is s-colourable and $|V(H)| \geq \frac{s}{t}|V(G)|$.)

But the situation is less clear when we look at *multi-colourings*. Here a (t, k)-*colouring* of a graph is an assignment of a k-subset of $\{1, 2, \ldots, t\}$ to each vertex such that adjacent vertices receive disjoint subsets.

In this talk we look at the following question: if a graph G is (t, k)-colourable, and we want to find a large (s, ℓ)-colourable induced subgraph of G (for some given (s, ℓ)), how large a part can we guarantee? Answering that question involves having a detailed look at Kneser graphs and related structures, and touches on several open problems in those areas.

Towards Stahl's Conjecture: Multi-Colouring of Kneser Graphs

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(This talk is based on joint work with Jan van den Heuvel.)

If a graph is n-colourable, then it obviously is n'-colourable for any $n' \geq n$. But the situation is not so clear when we consider *multi-colourings* of graphs. A graph is (n, k)-*colourable* if we can assign each vertex a k-subset of $\{1, 2, \ldots, n\}$, such that adjacent vertices receive disjoint subsets.

In this talk, we consider the following problem: if a graph is (n, k)-colourable, then for what pairs (n', k') is it also (n', k')-colourable? This question can be translated into a question regarding multi-colourings of Kneser graphs, for which Stahl formulated a conjecture in 1976. We present new results and discuss some observations that lead to simple proofs of some known cases of the conjecture.

Minimium color degree thresholds for rainbow subgraphs

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(This talk is based on joint work with Andrzej Czygrinow and Brendan Nagle.)

Let $G = (V, E)$ be a graph on n vertices and let $c : E \to \mathbb{N}$ be a coloring of the edges of G. The color degree $d^c(v)$ of a vertex $v \in V$ is the number of distinct colors that appear on the edges incident to v (i.e., $d^c(v) = |c^{-1}(\{e \in E : v \in e\})|$). We let $\delta^c(G) = \min_{v \in V}\{d^c(v)\}$ be the minimum color degree of G. In 2013, H. Li proved that if $\delta^c(G) \geq (n+1)/2$, then G contains a rainbow triangle and this is tight as witnessed by a properly edge-colored balanced bipartite graph. In this talk, we will explore generalizations and extensions of this result. In particular, for $\ell \geq 4$, we will discuss the minimum color degree threshold for the existence of a rainbow ℓ-clique.

From domination to isolation of graphs

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In 2017, Caro and Hansberg [6] introduced the isolation problem, which generalizes the domination problem. Given a graph G and a set $\mathcal{F}$ of graphs, the *$\mathcal{F}$-isolation number of G* is the size of a smallest subset D of the vertex set of G such that $G - N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. When $\mathcal{F}$ consists of a 1-clique, the $\mathcal{F}$-isolation number is the domination number. Caro and Hansberg [6] obtained many results on the $\mathcal{F}$-isolation number, and they asked for the best possible upper bound on the $\mathcal{F}$-isolation number for the case where $\mathcal{F}$ consists of a k-clique and for the case where $\mathcal{F}$ is the set of cycles. The solutions [1, 3] to these problems will be presented together with other results, including an extension of Chvátal's Art Gallery Theorem. Some of this work was done jointly with Kurt Fenech and Pawaton Kaemawichanurat.

[1] P. Borg, Isolation of cycles, *Graphs and Combinatorics* 36 (2020), 631–637.

[2] P. Borg, Isolation of connected graphs, arXiv:2110.03773.

[3] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques, *Discrete Mathematics* 343 (2020), paper 111879.

[4] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques II, *Discrete Mathematics*, in press.

[5] P. Borg and P. Kaemawichanurat, Domination and partial domination of maximal outerplanar graphs, arXiv:2002.06014.

[6] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, *FiloMath* 31:12 (2017), 3925–3944.

Separating Path Systems for the Complete Graph

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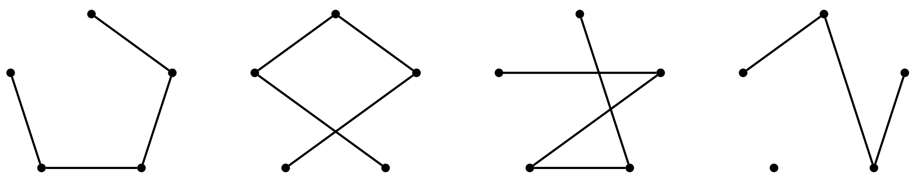
Let G be any graph and let $\mathcal{S}$ be a family of subsets of $E(G)$ such that for any (unordered) edges $e, e' \in E(G)$ there is some $P \in \mathcal{S}$ with $e \in P$ and $e' \notin P$. Then we say that $\mathcal{S}$ is a separating system for G and that P **separates** e and e'. If we also have the condition that every element of $\mathcal{S}$ is a path in G, then we call $\mathcal{S}$ a **separating path system** of G.

Below is an example of a separating path system for K_5. For any pair of edges from K_5, one of the paths below will contain exactly one of the two edges.



In general we wish to determine the smallest number of paths required for a separating path system of G, we use $f(G)$ to denote this value. Falgas-Ravry, Kittipassorn, Korándi, Letzter, and Narayanan [2] raised the question of determining $f(K_n)$, which will be the focus of this talk. The best known bounds for this are

$$n - 1 \leq f(K_n) \leq 2n + 4.$$

The lower bound uses a simple counting argument, while the upper bound in [1] uses a probabilistic argument. We give a construction improving the upper bound to

$$f(K_n) \leq \left(\frac{21}{16} + o(1)\right) n$$

in general, and to $f(K_n) \leq n$ for $n \leq 20$.

Our constructions use a concept we call generator paths, which reduces the problem to finding a single path which then generates a separating path system of n paths. We show that $f(K_n) \leq n$ if K_n contains a generator path. Such paths can be found by hand for $n \leq 20$. In general, we show that we can approximate a generator path for K_n and use a number of correcting paths to give a bound of $f(K_n) \leq (\frac{21}{16} + o(1))n$.

[1] J. Balogh, B. Csaba, R. Martin, and A. Pluhár. On the path separation number of graphs. *Discrete Applied Mathematics*, 213 (2016): 26-33.

[2] V. Falgas-Ravry, T. Kittipassorn, D. Korándi, S. Letzter, and B. Narayanan. Separating Path Systems. *Journal of Combinatorics*, 5.3 (2014): 335-354.

Monday 16:20, George Fox Lecture Theatre 1

Triangle saturated graphs with large minimum degree

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Given a graph H, we say that a graph G is H-saturated if G is maximally H-free, meaning G contains no copy of H but adding any new edge to G creates a copy of H. The general saturation problem is to determine $sat(n, H)$, the minimum number of edges in an H-saturated graph G on n vertices.

The special case when H is a triangle is straightforward - it is an easy exercise to show that $sat(n, K_3) = n-1$ for $n \geq 1$ and that the unique extremal graph is a star. Note that a star has many vertices of degree 1. One might ask what happens if we forbid such small degree vertices. We then have the more difficult problem of determining $sat(n, K_3, t)$, the minimum number of edges in a triangle saturated graph G on n vertices that additionally has minimum degree at least t.

Day [1] showed that for fixed t, $sat(n, K_3, t) = tn - c(t)$ for large enough n, where $c(t)$ is a constant depending on t. He proved the bounds $2^t t^{3/2} \ll c(t) \leq t^{t^{2t^2}}$. We show that the order of magnitude of $c(t)$ is given by $c(t) = \Theta\left(4^t/\sqrt{t}\right)$.

The order of magnitude of $c(t)$ turns out to be intimately related to Bollobás' celebrated Two Families Theorem. We end by presenting a conjectured generalisation of the Two Families Theorem, which, if proven, would allow one to extend these results from K_3 to general K_r.

[1] DAY, A. (2017). Saturated Graphs of Prescribed Minimum Degree. Combinatorics, Probability and Computing, 26(2), 201-207. doi:10.1017/S0963548316000377

Monday 16:45, George Fox Lecture Theatre 1

Complete subgraphs in a multipartite graph

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(This talk is based on joint work with Allan Lo and Yi Zhao.)

In 1975 Bollobás, Erdős, and Szemerédi asked the following question: given positive integers n, t, r with $2 \leq t \leq r-1$, what is the largest minimum degree $\delta(G)$ among all r-partite graphs G with parts of size n and which do not contain a copy of K_{t+1}? The $r = t+1$ case has attracted a lot of attention and was fully resolved by Haxell and Szabó, and Szabó and Tardos in 2006. In this talk we investigate the $r > t+1$ case of the problem, which has remained dormant for over forty years. We resolve the problem exactly in the case when $r \equiv -1 \pmod t$, and up to an additive constant for many other cases, including when $r \geq (3t-1)(t-1)$. Our approach utilizes a connection to the related problem of determining the maximum of the minimum degrees among the family of balanced r-partite rn-vertex graphs of chromatic number at most t.

Monday 11:15, George Fox Lecture Theatre 2

The sequence of prime gaps is graphic[1]

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(This talk is based on joint work with Harcos, Kharel, Maga, Mezei, Toroczkai.)

Let us call a simple graph on $n \geq 2$ vertices a **prime gap graph** if its vertex degrees are 1 and the first $n-1$ prime gaps. We show that such a graph exists for every large n, and in fact for every $n \geq 2$ if we assume the Riemann hypothesis. Moreover, an infinite sequence of prime gap graphs can be generated by the so-called degree preserving growth process.

This DPG process ([2]) is the iterative applications of **degree-preserving steps**, which can be described as follows: let G be a simple graph with degree sequence $\mathbf{D}$. In each step, a new vertex u joins the graph by removing a k-element matching of G followed by connecting u to the vertices incident to the k removed edges. The degree of the newly inserted vertex is $d = 2k$. The degree sequence of the newly generated graph is $\mathbf{D} \circ d$, that is, d is concatenated to the end of $\mathbf{D}$. The proofs are based on the following new graph theoretic results:

Theorem 1. (i) *Let $\mathbf{D} = (d_1, \ldots, d_n)$ be a sequence of positive integers such that $\|\mathbf{D}\|_1 = \sum_{\ell=1}^{n} d_\ell$ is even. Let $1 < p \leq \infty$ be a parameter. Assume that the following L^p-norm bound holds:*

$$\|2 + \mathbf{D}\|_p \leq n^{\frac{1}{2}+\frac{1}{2p}}.$$

Then degree sequence $\mathbf{D}$ satisfies the Erdős-Gallai condition, therefore it is graphic.
(ii) *Let G be any simple graph with degree sequence $\mathbf{D}$. Assume that $d \geq 2$ is an even integer satisfying*

$$4d^{1-\frac{1}{p}} \|\mathbf{D}\|_p \leq \|\mathbf{D}\|_1.$$

Then in G there exists a $d/2$-element matching, consequently $D \circ d$ is graphic.

On this basis we iteratively grow the infinite sequence of prime gap graphs, using DP-steps. The proof uses the Riemann hypothesis and new, serious analytical number theoretic results.

[1] P.L. Erdős - G. Harcos - S.R. Kharel - P. Maga - T.R. Mezei - Z. Toroczkai: The sequence of prime gaps is graphic, `arXiv:2205.00580` (2022), pp. 14.

[2] S. Kharel, T. R. Mezei, S. Chung, P. L. Erdős, Z. Toroczkai, Degree-preserving network growth, *Nature Physics* **18** (2022), 100–106. `DOI:10.1038/s41567-021-01417-7`

[1] This talk is based on [1].

On k-fold sums of integer sets Structure and irregularity

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For $k \in \mathbb{N}_0$ and a finite set $M \subseteq \mathbb{Z}$, we define the k-fold sum $kM := \{\sum_{i=1}^{k} x_i \mid x \in M^k\}$. Furthermore, we define the function $HF_M \colon \mathbb{N}_0 \to \mathbb{N}_0, k \mapsto |kM|$, which is a function of polynomial type, i.e. there are a rational polynomial p_M and a minimal non-negative integer k_0 such that $HF_M(k) = p_M(k)$ for every $k \geq k_0$. In the following, we are interested in a tight upper bound on k_0.

We investigate sets $M = \{m_0, m_1, \ldots, m_l\} \subseteq \mathbb{N}_0$ with $0 = m_0 < m_1 < \ldots < m_l$ and $\gcd(M) = 1$. These sets are called *normal* in [1].

By quantifying different ways in which the structure of multiple addition can be irregular for few summands, we determine an upper bound on k_0, which only depends on the maximal element m_l of M. In the proof, we use a theorem by Erdős, Ginzburg, and Ziv ([2], Theorem 2.5).

In a further attempt, we use the shape description of kM for large k, which was given by Nathanson in [2]. We generalise the description to smaller k and compare it to the structures we investigated before.

As an outlook, we discuss in which ways the stated results can be generalised to higher dimensional sets M.

[1] Vsevolod F Lev. Structure theorem for multiple addition and the frobenius problem. *journal of number theory*, 58(1):79–88, 1996.

[2] Melvyn B Nathanson. *Additive number theory: Inverse problems and the geometry of sumsets*, volume 165. Springer Science & Business Media, 1996.

On Combinaorial Number Theory: Sum Systems

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Let $m \in \mathbb{N}$, $\mathbf{n} = (n_1, \ldots, n_m) \in (\mathbb{N}+1)^m$ and $N = \prod_{j=1}^{m} n_j$. A collection of sets, $A_1, \ldots, A_m$, with cardinality $|A_j| = n_j$, is called a *sum system* if

$$\sum_{j=1}^{m} A_j = \{0, 1, 2, \ldots, N-1\},$$

where set addition is done by the Minkowski sum; $A + B = \{a + b | a \in A, b \in B\}$. The generation of consecutive integers, each term occurring uniquely, is a simple yet potent question to ask. The structure of these sum systems provide an insight into how multiplicative factors of N can be used to construct these additive systems. A core notion in this study is the following combinatorial object; we call

$$\Big((j_1, f_1), (j_2, f_2), \ldots, (j_L, f_L)\Big) \in \big(\{1, 2, \ldots, m\} \times (\mathbb{N}+1)\big)^L,$$

where $L \in \mathbb{N}$, a *joint ordered factorisation of* $\boldsymbol{n}$ if

$$\prod_{\ell \in \mathcal{L}_j} f_\ell = n_j, \quad \text{for} \quad \big(j \in \{1, \ldots, m\}\big),$$

with $\mathcal{L}_j := \{\ell \mid j_\ell = j\}$, and $j_\ell \neq j_{\ell-1}$ $\big(\ell \in \{2, \ldots, L\}\big)$.

This compact notation encodes the make-up of these additive systems. Alongside the notions of *arithmetic progressions*, $a\langle b\rangle := \{0, a, 2a, \ldots, (b-1)a\}$, and $F(\ell) := \prod_{s=1}^{\ell-1} f_s$, the joint ordered factorisation is utilised in the construction theorem for the sum system component sets by

$$A_j = \sum_{\ell \in \mathcal{L}_j} F(\ell)\langle f_\ell \rangle.$$

The enumeration of these objects, the study of the rich patterns within their structure and their connections to other combinatorial objects, such as difference families and necklaces, are why these systems are of great interest and why they are the focus of my thesis' investigation.

Graphs on lattices

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A (directed, not necessarily finite) graph $G = (V, E)$ can be viewed as a mapping $f : 2^V \to 2^V$ where $f(X) = N^{in}(X)$ is the in-neighbourhood of a subset of vertices $X \subseteq V$. A mapping $f : 2^V \to 2^V$ is the in-neighbourhood function of a graph if and only if it preserves arbitrary unions: $f(\bigcup_{X \in S} X) = \bigcup_{X \in S} f(X)$ for all $S \subseteq 2^V$. Mappings over a lattice that preserve arbitrary joins are called continuous: graphs can then be viewed as continuous mappings over Boolean lattices. In this talk, we shall import concepts and generalise results from graphs to so-called graphs on lattices, i.e. continuous mappings over a complete lattice L. First, we introduce strongly acyclic graphs and prove that they are exactly the graphs with a topological sort. Second, we introduce strongly acyclic tournaments and show that they are transitive. Third, we generalise the equivalence between finite topologies and pre-orders by recasting it as a result on the sets of fixed points of graphs. Fourth, Robert's theorem shows that a finite dynamical system with acyclic interaction graph converges to a unique fixed point. We finally introduce the interaction graph of a mapping $\phi : L \to L$ and generalise Robert's theorem, thus proving that a large class of mappings $\phi : L \to L$ converge to a unique fixed point.

Monday 15:55, George Fox Lecture Theatre 2

Learning Small Decision Trees for Data of Low Rank-Width

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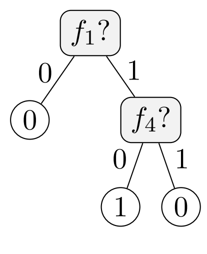
(This talk is based on joint work with Eduard Eiben, Sebastian Ordyniak, Giacomo Paesani and Stefan Szeider.)

A *classification instance* consists of a finite set E of examples (also called *feature vectors*). Each example $e \in E$ is a function $e : feat(E) \to \{0, 1\}$ which determines whether the feature f is true or false for e. The set E is given as a partition $E^+ \uplus E^-$ into *positive* and *negative* examples. For instance, examples could represent medical patients and features diagnostic tests; a patient is positive or negative, corresponding to whether they have been diagnosed with a certain disease or not. The *incidence graph* $G(E)$ is the bipartite graph with features and examples being the vertices, where an example is adjacent to all features that are true for it.

A *decision tree* is a rooted tree binary tree whose internal nodes are features (with one child being negative and the other positive) and whose leaves are either 0 or 1, corresponding to negative and positive, respectively. A decision tree *classifies* a classification instance if we can correctly decide whether the example is positive or negative by going from the root to the leaves, always choosing the positive or negative child of a node if the example has that feature, or not, respectively.

E	f_1	f_2	f_3	f_4
$e_1 \in E^-$	0	0	1	0
$e_2 \in E^-$	0	0	1	1
$e_3 \in E^-$	0	1	1	0
$e_4 \in E^-$	1	1	0	0
$e_5 \in E^+$	1	0	0	1
$e_6 \in E^+$	1	0	1	1



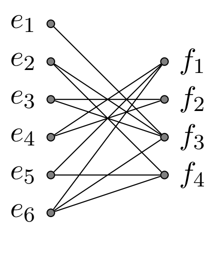


Figure 1: A classification instance $E = E^+ \uplus E^-$ with six examples and four features, a decision tree with 5 nodes that classifies E, the incidence graph $G(E)$.

Finding a decision tree of smallest size is an NP-hard problem. We show that we can solve the problem in $f(k)|E|^p$ operations, where k is the rank-width of the incidence graph, f is a computable function independent of $|E|$ and p is a constant.

The talk will not assume any prior knowledge of graph widths, decision trees or parameterized complexity. I will explain the intuition behind how the algorithm works, and how to go about constructing such dynamic programming algorithms on graphs of bounded widths.

Large independent sets in Markov random graphs

Akshay Gupte

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(This talk is based on joint work with Yiran Zhu.)

Finding the maximum size of an independent set in a graph $G = (V, E)$, called the stability number $\alpha(G)$, is a difficult combinatorial problem that is in general NP-hard to approximate within factor $|V|$ [Hås99]. There have been numerous studies on bounding $\alpha(G)$ asymptotically for Erdös-Rényi binomial random graphs [BE76, Fri90, GM75, Mat76], and more so about the chromatic number $\chi(G)$, which yields a lower bound on $\alpha(G)$ and its concentration [Bol88, COPS08, Hec18, Hec21, Luc91, McD90]. We continue this line of work on asymptotic analysis of $\alpha(G)$ but initiate it on a new class of random graphs for which the Erdös-Rényi graphs are a boundary condition. The edges in our random graph are generated dynamically using a Markov process. Given n, p and a decay parameter $\delta \in (0, 1]$, starting from the singleton graph $(\{v_1\}, \emptyset)$, a graph $G^{\delta}_{n,p}$ having n vertices is generated in $n - 1$ iterations where at each iteration $t \geq 2$, the vertex v_t is added to the graph and edges (v_i, v_t) for $1 \leq i \leq t - 1$ are added as per a Bernoulli r.v. X^t_i. The success probability $\mathsf{Pr}\{X^t_i = 1\}$ is equal to p for $i = 1$ and for $i \geq 2$, it is independent of the values of $\{X^t_1, \ldots, X^t_{i-2}\}$ and is equal to $\mathsf{Pr}\left\{X^t_{i-1} = 1\right\}$ when $X^t_{i-1} = 0$ and equal to $\delta\,\mathsf{Pr}\left\{X^t_{i-1} = 1\right\}$ when $X^t_{i-1} = 1$. It follows that $G^1_{n,p}$ is the binomial random graph $G_{n,p}$, and so the Erdös-Rényi model is a limiting case of our model.

Our main theorem is that the size of the independent sets in $G^{\delta}_{n,p}$ grows at least as rapidly as the number of primes less than n. In particular, let $\pi(n)$ denote the prime-counting function.

Theorem 1. *For every $\epsilon > 0$ and $\delta \in (0, 1)$, we have w.h.p. that* $\alpha(G^{\delta}_{n,p}) \geq \dfrac{2+\epsilon}{1-\delta}\pi(n)$.

To prove this theorem, we establish that the average vertex degree in $G^{\delta}_{n,p}$, which we denote by $d(G^{\delta}_{n,p})$, scaled by a logarithmic factor concentrates to 2. For this concentration result, we use Chebyshev's inequality. Due to the absence of independence structure between r.v.'s, we cannot apply Chernoff/Hoeffding-type inequalities, and use of martingale tail inequalities also does not help. For fixed p, this theorem shows $G^{\delta}_{n,p}$ to be more sparse than $G_{n,p}$ in terms of the number of edges.

On the upper-bounding side, we provide a tight constant $c < 1$ that bounds $\alpha(G^{\delta}_{n,p}) \leq c\,n$.

Theorem 2. *For every $\delta \in (0, 1)$, we have w.h.p. that* $\alpha(G^{\delta}_{n,p}) \leq \left(e^{-\delta} + \frac{\delta}{10}\right) n$.

Since all of our analysis heavily depends on $\delta < 1$, our results don't generalise those known for the Erdös-Rényi graph ($\alpha(G_{n,p}) \approx 2\log_{\frac{1}{1-p}} n$ for fixed p), which also indicates that a phase transition occurs in our random graph model at the boundary value $\delta = 1$.

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[GM75] Geoffrey R Grimmett and Colin JH McDiarmid. On colouring random graphs. *Mathematical Proceedings of the Cambridge Philosophical Society*, 77(2):313–324, 1975.

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[McD90] Colin McDiarmid. On the chromatic number of random graphs. *Random Structures & Algorithms*, 1(4):435–442, 1990.

Enumerating pattern-avoiding inversion sequences: an algorithmic approach based on generating trees

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(This talk is based on joint work with Toufik Mansour.)

An *inversion sequence* of length n is an integer sequence $e = e_1 \cdots e_n$ such that $0 \leq e_i < i$ for each $0 \leq i \leq n$. We use I_n to denote the set of inversion sequences of length n. There is a bijection between I_n and S_n, the set of permutations of length n. Given any word τ of length k over the alphabet $[k] := \{0, 1, \cdots, k-1\}$, it is said that an inversion sequence $e \in I_n$ contains the pattern τ if there is a subsequence of length k in e that is order isomorphic to τ; otherwise, e avoids the pattern τ. For instance, $e = 01102321 \in I_8$ avoids the pattern 0000 because there is no subsequence $e_i e_j e_k e_l$ of length four in e with $i < j < k < l$ and $e_i = e_j = e_k = e_l$. On the other hand, $e = 01102321$ contains the patterns 010 and 000 because it has subsequences $-1---3-1$ or $----232-$ order isomorphic to 010, and subsequence $-11----1$ order isomorphic to 000. For a given pattern τ, we let $I_n(\tau)$ denote the set of all τ-avoiding inversion sequences of length n. Similarly, for a given set of patterns B, we set $I_n(B) = \cap_{\tau \in B} I_n(\tau)$. Pattern-avoiding inversion sequences systematically were studied first by Mansour and Shattuck [2] for the patterns of length three with non-repeating letters, and by Corteel et al. [1] for repeating and non-repeating letters. There are basically thirteen patterns of length three up to order isomorphism $\mathcal{P} = \{000, 001, 010, 100, 011, 101, 110, 021, 012, 102, 120, 201, 210\}$. We provide an algorithmic approach based on generating trees for enumerating the pattern-avoiding inversion sequences. By using this approach, we determine the generating trees for the pattern-classes $I_n(000, 021), I_n(100, 021)$, $I_n(110, 021)$, $I_n(102, 021)$, $I_n(100, 012)$, $I_n(011, 201)$, $I_n(011, 210)$ and $I_n(120, 210)$. Then we obtain generating functions of each class, and find enumerating formulas. Lin and Yan [3] studied the classification of the Wilf-equivalences for inversion sequences avoiding pairs of length-three patterns and showed that there are 48 Wilf classes among 78 pairs. We solve six open cases for such pattern classes.

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Monday 11:40, George Fox Lecture Theatre 3

Asymptotic Behaviour of Mesh Pattern Containment

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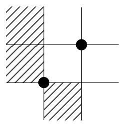
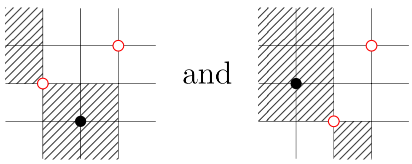
(This talk is based on joint work with Dejan Govc.)

A *mesh pattern* is a pair (π, P), where π is a permutation and P is a set of coordinates in a square grid. For example, $p = (12, \{(0,1), (0,2), (1,0)\})$ is a mesh pattern, which we can depict as

 .

We say a permutation τ *contains* a mesh pattern (π, P) if there is an occurrence of π in τ, in the traditional permutation pattern sense, such that when mapping (π, P) onto this occurrence in the picture of τ no dot of τ appears in a shaded region. For example, the permutation 213 does not contain p (from above) because there are two occurrences of 12 in 213, shown by the hollow red points below, but in both cases the other dot is within one of the shaded regions:

 .

In this talk we present some results on the proportion of permutations containing certain mesh patterns as n grows large, that is, the limit

$$\lim_{n\to\infty} \frac{s_n(p)}{n!}, \tag{1}$$

where p is a mesh pattern and $s_n(p)$ is the number of permutations of length n containing p. We present some formulas for (1) when p is a mesh pattern with entire rows and columns shaded and for particular mesh patterns of length four. An important consequence of these results is that the limit can take a wide range of values between 0 and 1, which is not true in the traditional permutation patterns setting.

[1] Dejan Govc and Jason P. Smith. Asymptotic behaviour of the containment of certain mesh patterns. *Discrete Mathematics*, 345(5):112813, 2022.

Touching representations by comparable boxes

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(This talk is based on joint work with Zdeněk Dvořák, Daniel Gonçalves, Abhiruk Lahiri and Torsten Ueckerdt.)

Two boxes in $\mathbb{R}^d$ are comparable if one of them is a subset of a translation of the other. The comparable box dimension of a graph G is the minimum d such that G can be represented as a touching graph of comparable axis-aligned boxes in $\mathbb{R}^d$. Having finite comparable box dimension implies a number of nice graph properties, which leads us to consider which graphs have such geometric representations. In this talk, we show that comparable box representations behave well under several common operations on graphs. This leads to a proof that every proper minor-closed class of graphs has finite comparable box dimension.

On a new family of algebraically defined graphs

Vladislav Taranchuk

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(This talk is based on joint work with Felix Lazebnik.)

Over the past few decades, algebraically defined graphs have gained a lot of attention due to their applications to Turan type problems in graph theory and their connections to finite geometries. In this talk, we discuss how the algebraically defined graphs have been used to tackle a long standing question regarding the existence of new generalized quadrangles. Furthermore, we demonstrate a new family of algebraically defined graphs whose existence implies that there are potentially many new families of graphs yet to be studied that may provide a new generalized quadrangle. This talk is based on joint work with Felix Lazebnik (University of Delaware).

What is a (combinatorial) sandpile?

Thomas Selig

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Xi'an Jiaotong-Liverpool University

The sandpile model is a dynamic process on a graph G. At each unit of time, a grain of sand is added to a randomly selected vertex of G. When this causes the number of grains at that vertex to exceed a certain threshold (usually its degree), that vertex is said to be unstable, and topples, sending grains to its neighbours in G. Of central interest in sandpile model research are the recurrent states, those that appear infinitely often in the long-time running of the model.

One recent fruitful research direction in sandpile research concerns the combinatorial study of these recurrent states on specific graph families with high degree of symmetry. In these cases, the additional structure of the underlying graph allows us to establish bijections to related combinatorial objects which can be more easily calculated. The seminal example is the bijection to parking functions in the complete graph case [1], while results of this type have also been discovered for complete bipartite graphs [3], complete split graphs [2], wheel and fan graphs [4], and many others.

In this talk, we will focus on the complete graph and wheel graph cases. We show that in the wheel graph case, the recurrent states of the sandpile model are in bijection with subgraphs of the cycle. Through these two illustrative examples, we will consider the intriguing possibility of a "meta-theorem" relating combinatorial sandpiles to decorated combinatorial structures.

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[2] M. Dukes. The sandpile model on the complete split graph, Motzkin words, and tiered parking functions. *J. Comb. Theory, Ser. A*, 180:15, 2021.

[3] M. Dukes and Y. Le Borgne. Parallelogram polyominoes, the sandpile model on a complete bipartite graph, and a q, t-Narayana polynomial. *J. Comb. Theory, Ser. A*, 120(4):816–842, 2013.

[4] T. Selig, Combinatorial aspects of sandpile models on wheel and fan graphs: subgraphs of cycles and lattice paths, arXiv:2202.06487 [math.CO], 2022.

Software for finding and classifying cliques

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Queen Mary University of London

I will describe a new hybrid GAP [1]/GRAPE [3]/C program for determining the cliques with given vertex-weight sum in a graph whose vertices are weighted with non-zero d-vectors of non-negative integers. This program is designed to exploit graph symmetry and may be used for parallel computation on an HPC cluster, such as the QMUL Apocrita cluster [2]. Some research applications will be presented.

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[3] L. H. Soicher, The GRAPE package for GAP, Version 4.8.5, 2021. https://gap-packages.github.io/grape

Neighbour-transitive codes in Kneser graphs

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Faculty of Mathematics, University of Rijeka

(This talk is based on joint work with Dean Crnković, Nina Mostarac and Andrea Švob.)

A *code* is a subset of the vertex set of a graph. Classically codes have been studied in the Hamming and Johnson graphs. Here we consider codes in odd and Kneser graphs, whose vertices are subsets of an underlying set Ω. A code C is *neighbour-transitive* if the automorphism group $\mathrm{Aut}(C)$ of the code acts transitively on the code, and also on the set of vertices at distance one from the code. We give several results in the direction of a classification of neighbour-transitive codes in Kneser graphs. First, if C is a neighbour-transitive code in a Kneser graph and $\mathrm{Aut}(C)$ acts intransitively on Ω then we classify the parameters of C and give several examples. If C is a neighbour-transitive code in an odd graph and $\mathrm{Aut}(C)$ acts imprimitively on Ω then we again classify the parameters of C and give an example in each case. We provide a structural result for the case that C is a code in a Kneser graph that is not odd and C has minimum distance at least 3. Finally, we give a full classification of 2-*neighbour-transitive* codes with minimum distance at least 5 in Kneser graphs.

Monday 11:15, George Fox Lecture Theatre 5

Recursively Counting Flows in Embedded Graphs

Maya Thompson

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Royal Holloway, University of London

(This talk is based on joint work with Iain Moffatt.)

The number of nowhere-zero flows in a graph is a well understood problem and easily obtained as a specialisation of the Tutte polynomial. Additionally, using the recursive form of the Tutte polynomial we can obtain a recursion on the number of nowhere-zero flows.

In recent work by Goodall, Litjens, Regts and Vena they found a polynomial that also counts the number of local flows for graphs embedded in a surface. By extending their polynomial to the family of non-cellularly embedded graphs in pseudo-surfaces, we can express their polynomial as a recursion which naturally extends to a recursive way to count flows in embedded graphs.

In this talk, I will show how the recursion works and use the relationship between the topology of the surface and the number of flows to provide some intuition behind the recursion.

A critical group for embedded graphs: working with maps

Iain Moffatt

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Royal Holloway, University of London

(This talk is based on joint work with Criel Merino and Steven D. Noble .)

Critical groups are finite Abelian groups associated with graphs. They are well-established in combinatorics, closely related to the graph Laplacian and arise in several contexts such as chip firing and parking functions. The order of the critical group of a connected graph is equal to its number of spanning trees, a fact equivalent to Kirchhoff's Matrix–Tree Theorem.

How should we define critical groups for graphs embedded in surfaces, rather than for graphs in the abstract? This is the first of two talks in which we answer this question. (Steve Noble will give the second talk.)

In this talk the emphasis will be on topological graph theory, and the interactions of the problem with Chumtov's partial-duals, one-face subgraphs, and a Matrix–quasi-Tree Theorem of Macris and Pule.

Both talks will stand alone, so don't worry if you miss either one!

A critical group for embedded graphs: working with delta-matroids

Steven Noble

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Birkbeck, University of London

(This talk is based on joint work with Criel Merino, Iain Moffatt.)

Critical groups are finite Abelian groups associated with graphs. They are well-established in combinatorics, closely related to the graph Laplacian and arise in several contexts such as chip firing and parking functions. The order of the critical group of a connected graph is equal to its number of spanning trees, a fact equivalent to Kirchhoff's Matrix–Tree Theorem.

How should we define critical groups for graphs embedded in surfaces, rather than for graphs in the abstract? This is the second of two talks in which we answer this question. (The first talk was given by Iain Moffatt.)

In the first talk topological graph theory suggested a way to define a critical group for graphs embedded in orientable surfaces. But it is far from obvious that this definition works. In this talk we reframe our construction in terms of regular delta-matroids and use this more general setting to finally determine the definition of a critical group.

Both talks will stand alone, so don't worry if you miss either one!

Monday 15:30, George Fox Lecture Theatre 5

Characterising Global Rigidity in non-Euclidean Normed Planes via Matroid Connectivity

John Hewetson

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Lancaster University

(This talk is based on joint work with Sean Dewar (RICAM), Tony Nixon (Lancaster).)

A framework (G, p) is an ordered pair where G is a graph and p maps the vertices of G to some normed space. In the 1990s, Hendrickson [1] gave necessary conditions for a generic framework to be globally rigid in d-dimensional Euclidean space. Connelly proved that Hendrickson's conditions are insufficient when $d \geq 3$, but in 2005 they were shown to be sufficient when $d = 2$. This result combined work by Connelly [2] with a construction of a family of graphs by Jackson and Jordán [3]. In particular, Jackson and Jordán gave a construction of those graphs for which the corresponding $(2, 3)$-sparsity matroid is connected. More recently, attention has turned to considering frameworks realised in non-Euclidean normed spaces. In this talk we present our construction of those graphs for which the corresponding $(2, 2)$-sparsity matroid is connected, and use this to give a characterisation of globally rigid frameworks in analytic (non-Euclidean) normed planes.

[1] Bruce Hendrickson. Conditions for unique graph realizations. *SIAM Journal of Computing*, 21(1):65–84, 1992.

[2] Robert Connelly. Generic Global Rigidity. *Discrete & Computational Geometry. Algorithms*, 33:549–563, 2005.

[3] Bill Jackson and Tibor Jordán. Connected rigidity matroids and unique realizations of graphs. *Journal of Combinatorial Theory, Series B*, 94(1):1–29, 2005.

Monday 15:55, George Fox Lecture Theatre 5

Flexibility of Penrose frameworks

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Department of Applied Mathematics, Faculty of Information Technology,
Czech Technical University in Prague

(This talk is based on joint work with Sean Dewar.)

A framework, which is a (possibly infinite) graph together with a realization of its vertices in the plane, is called flexible if it can be continuously deformed while preserving the distances between adjacent vertices. The existence of a flexible framework for a given graph is characterised by the existence of a so called *NAC-coloring* — a surjective edge coloring by red and blue such that each cycle is either monochromatic, or contains at least two red and two blue edges.

In this talk, we focus on infinite frameworks obtained as 1-skeleta of parallelogram tilings. We brace some of the parallelograms, namely, they are not allowed to change their shape during a flex. We show that such a structure is flexible if and only if the graph admits a special type of NAC-coloring, called *cartesian*. Moreover, if this framework is n-fold rotationally symmetric, we can again decide its flexibility by the existence a cartesian NAC-coloring invariant under the symmetry. In particular, we can apply these results to frameworks obtained from (5-fold symmetric) Penrose tilings, see Figure 1.

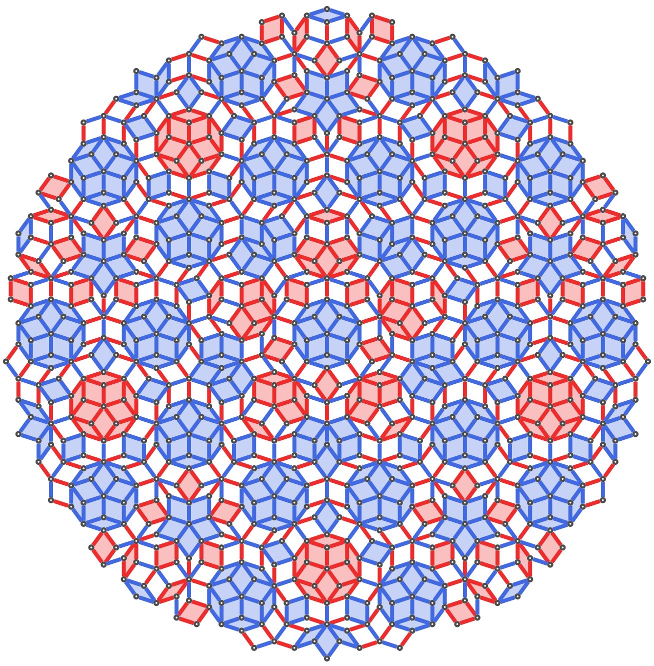

Figure 1: A finite piece of an infinite 5-fold symmetric Penrose tiling with a NAC-coloring certifying its flexibility: the filled rhombi preserve their shapes along the flex.

Monday 16:20, George Fox Lecture Theatre 5

Symmetric contact systems of segments, pseudotriangulations and inductive constructions for corresponding surface graphs.

James Cruickshank

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National University of Ireland, Galway

(This talk is based on joint work with Bernd Schulze (Lancaster University).)

We prove symmetric analogues of two well known theorems in combinatorial geometry. The first is a result of Thomassen concerning contact graphs of collections of line segments in the plane (see Section 2.2 of [3] for a description of this result) . The second is due to Haas et al. and characterises graphs that have embeddings as pointed pseudotriangulations in the plane (see [2]). The symmetric setting gives rise naturally to graphs that are embedded in non-planar surface. The main technical result that we use is a new inductive construction of an appropriate class of surface graphs that is common to both situations.

[1] J. Cruickshank, B. Schulze. Symmetric contact systems of segments, pseudotriangulations and inductive constructions for corresponding surface graphs. 2021 preprint available at: https://arxiv.org/abs/2006.10519

[2] R. Haas, D. Orden, G. Rote, F. Santos, B. Servatius, H. Servatius, D. Souvaine, I. Streinu, and W. Whiteley. Planar minimally rigid graphs and pseudo-triangulations. *Comput. Geom.*, 31(1-2):31–61, 2005.

[3] P. Hliněný. Classes and recognition of curve contact graphs. *J. Combin. Theory Ser. B*, 74(1):87–103, 1998.

BOUNDS RELATED TO THE EDGE-LIST CHROMATIC AND TOTAL CHROMATIC NUMBERS OF A SIMPLE GRAPH

A. J. W. Hilton

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University of Reading

(This talk is based on joint work with R. Mary Jeya Jothi and M. Henderson.)

We show that for a simple graph G, $c'(G) \leq \Delta(G) + 2$ where $c'(G)$ is the choice index (or edge-list chromatic number) of G, and $\Delta(G)$ is the maximum degree of G.

As a simple corollary of this result, we show that the total chromatic number $\chi_T(G)$ of a simple graph satisfies the inequality $\chi_T(G) \leq \Delta(G) + 4$ and that the total choice number $c_T(G)$ also satisfies this inequality.

We also relate these bounds to the Hall index and the Hall condition index of a simple graph, and to the total Hall number and the total Hall condition number of a simple graph.

Monochromatic linear forests

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Miami University

(This talk is based on joint work with András Gyárfás and Gabor Sárközy.)

We prove that in every r-coloring of K_n there is a monochromatic linear forest on $3\left\lfloor\frac{n}{r+2}\right\rfloor$ vertices, which is best possible when $r+2$ divides n. This generalizes the 2-color case which was solved by Burr and Roberts in 1974. One of the main ingredients in our proof is an estimate on the size of non-uniform covering designs.

Monochromatic components with many edges

Mykhaylo Tyomkyn

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Charles University

(This talk is based on joint work with David Conlon and Sammy Luo.)

Given an r-edge-coloring of the complete graph K_n, what is the largest number of edges in a monochromatic connected component? This natural question has only recently received the attention it deserves, with work by two disjoint subsets of the authors resolving it for the first two special cases, when $r = 2$ or 3. Here we introduce a general framework for studying this problem and apply it to fully resolve the $r = 4$ case, showing that such a coloring always yields a monochromatic component with at least $\frac{1}{12}\binom{n}{2}$ edges, where the constant $\frac{1}{12}$ is optimal only when the coloring matches a certain construction of Gyárfás.

Balancing connected colourings of graphs

Youri Tamitegama

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University of Oxford

(This talk is based on joint work with Freddie Illingworth, Emil Powierski, Alex Scott.)

In a seminal result on subgraph packing, Tutte [2] and Nash-Williams [3] characterise finite graphs containing two edge-disjoint spanning trees. It is natural to ask whether such graphs admit packings with additional properties, such as 'balanced' packings. Specifically, does a graph which is precisely the union of two edge-disjoint spanning trees have a blue/red edge-colouring such that the colour degrees at each vertex differ by at most a constant c? The first finite upper bound on c is due to Hörsch [1]. In this talk, we sketch the proof of an improved bound of $c \leq 4$. If time allows, we will discuss extending this bound to blue/red connected colourings of arbitrary graphs containing two edge-disjoint spanning trees.

[1] F. Hörsch, Globally balancing spanning trees, *arXiv:2110.13726*, 2021

[2] W. T. Tutte, On the problem of decomposing a graph into n connected factors, *Journal of the London Mathematical Society*, **1**, (1):221–230, 1961

[3] C. Nash-Williams, Edge-disjoint spanning trees of finite graphs, *Journal of the London Mathematical Society*, **1**, (1):445–450, 1961

Tuesday 10:30, George Fox Lecture Theatre 2

(Random) trees of intermediate volume growth exist

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(This talk is based on joint work with George Kontogeorgiou.)

For every sufficiently nice increasing function $g\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ that grows at least linearly and at most exponentially we construct a tree T with uniform volume growth $g(r)$, that is,

$$C_1 \cdot g(r/4) \,\leq\, |B_v(r)| \,\leq\, C_2 \cdot g(4r), \quad \text{for all } r \geq 0,$$

where $B_v(r)$ denotes the ball of radius r centered at a vertex v. In particular, this yields examples for trees of uniform intermediate volume growth.

This constructions can be extended to yield unimodular random trees of uniform intermediate growth (answering a question by Benjamini), as well as triangulations of the plane with the same wide range of growth behaviors.

Subgraphs of Semi-random Graphs

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(This talk is based on joint work with Trent Marbach, Paweł Prałat and Andrzej Ruciński.)

The semi-random graph process can be thought of as a one player game. Starting with an empty graph on n vertices, in each round a random vertex u is presented to the player, who chooses a vertex v and adds the edge uv to the graph. Given a graph property, the objective of the player is to force the graph to satisfy this property in as few rounds as possible.

We will consider the property of constructing a fixed graph G as a subgraph of the semi-random graph. Ben-Eliezer, Hefetz, Kronenberg, Parczyk, Shikhelman and Stojacovič proved that the player can asymptotically almost surely construct G given $n^{1-1/d}\omega$ rounds, where ω is any function tending to infinity with n and d is the degeneracy of the graph G. We prove a matching lower bound. I will talk about this result, and also discuss a generalisation of our approach to semi-random hypergraphs. I will finish with some open questions.

Maximum running times for graph bootstrap percolation processes

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(This talk is based on joint work with David Fabian and Tibor Szabó.)

Given a fixed graph H and an n-vertex graph G the *H-bootstrap percolation process* of H on G is defined to be the sequence of graphs G_i, $i \geq 0$ which starts with $G_0 := G$ and in which G_{i+1} is obtained from G_i by adding every edge that completes a copy of H. This process is an example of a cellular automata and has been extensively studied since being introduced by Bollobás [2] in 1968. Recently, Bollobás raised the question of determining the maximum running time of this process, over all choices of n-vertex graph G. Here, the running time of the process is number of steps t the process takes before stabilising, that is, when $G_t = G_{t+1}$. Recent papers of Bollobás–Przykucki–Riordan–Sahasrabudhe [3], Matzke [4] and Balogh–Kronenberg–Pokrovskiy–Szabó [1] have addressed the case when H is a clique, and determined the asymptotics of this maximum running time for all cliques apart from K_5. Here, we initiate the study of the maximum running time for other graphs H and provide a survey of our new results in this direction. We study several key examples, giving precise results for trees and cycles, and giving general results towards understanding how the maximum running time of the H-bootstrap percolation process depends on properties of H, in particular exploring the relationship between this graph parameter and the degree sequence of H. Many interesting questions remain and along the way, we indicate some directions for future research.

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[2] B. Bollobás. Weakly k-saturated graphs. In *Beiträge zur Graphentheorie (Kolloquium, Manebach, 1967)*, pages 25–31, 1968.

[3] B. Bollobás, M. Przykucki, O. Riordan, and J. Sahasrabudhe. On the maximum running time in graph bootstrap percolation. *Electronic Journal of Combinatorics*, 24(2), 2017.

[4] K. Matzke. The saturation time of graph bootstrap percolation. *arXiv preprint arXiv:1510.06156*, 2015.

Size-Ramsey numbers of graphs with maximum degree three

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(This talk is based on joint work with Nemanja Draganić.)

The size-Ramsey number $\hat{r}(H)$ of a graph H is the smallest number of edges a (host) graph G can have, such that for any red/blue coloring of G, there is a monochromatic copy of H in G. Recently, Conlon, Nenadov and Trujić showed that if H is a graph on n vertices and maximum degree three, then $\hat{r}(H) = O(n^{8/5})$, improving upon the bound of $n^{5/3+o(1)}$ by Kohayakawa, Rödl, Schacht and Szemerédi. In this work, we show that $\hat{r}(H) \leq n^{3/2+o(1)}$. While the previously used host graphs were vanilla binomial random graphs, we prove our result using a novel host graph construction. We also discuss why our bound is a natural barrier for the existing methods.

Ramsey equivalence for asymmetric pairs

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(This talk is based on joint work with Simona Boyadzhiyska, Dennis Clemens, and Jonathan Rollin.)

A graph F is a Ramsey graph for a pair (G, H) of graphs if any red/blue-coloring of the edges of F yields a copy of G with all edges colored red or a copy of H with all edges colored blue. Two pairs of graphs are called Ramsey equivalent if they have the same collection of Ramsey graphs. The symmetric setting, that is, the case $G = H$, received considerable attention which constituted the open question whether there are connected graphs G and G' such that (G, G) and (G', G') are Ramsey equivalent. We study the asymmetric version of this question and identify several non-trivial families of Ramsey equivalent pairs of connected graphs.

Certain pairs of stars provide a first, albeit trivial, example of Ramsey equivalent pairs of connected graphs. Our results characterize all Ramsey equivalent pairs of stars. The rest of the work focuses on pairs of the form (T, K_t), where T is a tree and K_t is a complete graph. We show that, if T belongs to a certain family of trees, including all non-trivial stars, then (T, K_t) is Ramsey equivalent to a family of pairs of the form (T, H), where H is obtained from K_t by attaching smaller disjoint cliques to some of its vertices. On the other hand, we prove that for many other trees T, including all odd-diameter trees, (T, K_t) is not equivalent to any such pair, even not to the pair $(T, K_t \cdot K_2)$, where $K_t \cdot K_2$ is a complete graph K_t with a single edge attached.

Fixed-point cycles: extremal combinatorics meets social choice theory

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(This talk is based on joint work with Benjamin Aram Berendsohn and László Kozma.)

Given an edge-labeling of the complete bidirected graph K_n with functions from $[d]$ to itself, we call a cycle in K_n a *fixed-point cycle* if composing the labels of its edges results in a map that has a fixed point; the labeling is *fixed-point-free* if no fixed-point cycle exists. In this talk, we will consider the following question: for a given d, what is the largest value of n for which there exists a fixed-point-free edge-labeling of K_n with functions from $[d]$ to itself? This question was raised in a recent paper of Chaudhury, Garg, Mehlhorn, Mehta, and Misra studying a problem in social choice theory. As it turns out, it is also closely related to the problem of finding zero-sum cycles in edge-labeled digraphs, recently considered by Alon and Krivelevich and by Mészáros and Steiner. We will discuss these connections and present some new results related to both problems.

Tuesday 11:45, George Fox Lecture Theatre 3

Ramsey theory on homogeneous structures

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Ramsey theory on relational structures has been investigated ever since Ramsey proved his seminal theorem for colorings of k-sized sets of natural numbers. While a multitude of classes of finite structures have been shown to possess the Ramsey property, such as finite linear orders and finite ordered graphs, analogues for infinite structures have proven more elusive: Initiated by Sierpiński in the 1930's, it was not until D. Devlin's work in 1979 that the Ramsey theory of the rationals as a linearly ordered structure was completely understood; the Ramsey theory of the Rado graph was only completed in 2006 by work of Laflamme, Sauer, and Vuksanovic. Methods for Ramsey theory on finite structures are generally not sufficient for discovering Ramsey properties of their infinite homogeneous counterparts, i.e., Fraïssé limits, because upon well-ordering a homogeneous structure, the interplay between this ordering and the relations persists in every isomorphic substructure leading to unavoidable colorings with many colors.

This talk follows up on the speaker's talk at the 2019 BCC on the Ramsey theory of Henson graphs. Methods discussed then (using coding trees, set theory, and some model theoretic ideas) have paved the way to a fruitful expansion of results for various classes of homogeneous structures, including binary relational free-amalgamation classes and the generic partial order. This talk will be a condensed version of the speaker's 2022 ICM talk, providing an overview of the current state of Ramsey theory of homogeneous structures, built on works of various author combinations from among Balko, Barbosa, Chodounský, Coulson, El-Zahar, Erdős, Hajnal, Hubička, Komjáth, Konečný, Laflamme, Larson, Mašulović, Nešetřil, Nguyen Van Thé, Patel, Pósa, Rödl, Sauer, Vena, Zucker, and the speaker.

Position sets in graphs

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(This talk is based on joint work with E. Thomas, U. Chandran, G. Di Stefano, G. Erskine, N. Salia, C. Tompkins, S. Klavžar, P. Neethu, M. Thankachy.)

The *general position problem* for graphs was inspired by a puzzle of Dudeney and the general position subset selection problem in discrete geometry; it asks for the largest set S of vertices in a graph G such that no shortest path of G contains ≥ 3 vertices of S. In this talk, we shall discuss some extremal questions for variants of this problem, including *equilateral sets* (sets of vertices at equal distance) and *monophonic position sets* (sets of vertices of a graph G such that no induced path of G contains ≥ 3 vertices of S), including some intriguing connections to Turán and Ramsey problems.

Tuesday 10:55, George Fox Lecture Theatre 5

h^*-VECTORS OF EDGE POLYTOPES AND CONNECTIONS TO THE GREEDOID POLYNOMIAL

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(This talk is based on joint work with Tamás Kálmán.)

The edge polytope of a directed graph G is defined as

$$\mathcal{Q}_G = \mathrm{Conv}\{\, \mathbf{1}_h - \mathbf{1}_t \mid \overrightarrow{th} \in E(G)\,\} \subset \mathbb{R}^{V(G)}.$$

An interesting special case is if the graph G is bidirected; that is, it is obtained from an undirected graph by substituting each edge with two oppositely directed edges. In this case, the polytope is called a symmetric edge polytope. The volume and h^*-polynomial of the symmetric edge polytope recently received considerable interest due to their nice properties and relationship to the Kuramoto model in physics.

The dimension of the edge polytope is typically $|V(G)|-1$, but it can be $|V(G)|-2$. The latter is true exactly for those digraphs, where each cycle has the same number of edges pointing in the two cyclic directions. We call these digraphs semi-balanced. The edge polytopes of semi-balanced digraphs appear as facets of symmetric edge polytopes.

We give various formulas for the h^*-polynomials of symmetric edge polytopes, and edge polytopes of semi-balanced digraphs as generating functions of certain activities for certain spanning trees. We present an open question about which formulas of this type are true.

Also, we show that the greedoid polynomial of a planar Eulerian branching greedoid is equivalent to the h^*-polynomial of the edge polytope of a semi-balanced digraph. Indeed, it turns out that for a planar Eulerian digraph, its dual is a semi-balanced digraph, and the greedoid polynomial of the branching greedoid is equivalent to the h^*-polynomial of the edge polytope of the dual graph. This result can be generalized to any Eulerian digraph if one suitably defines the edge polytope of a regular oriented matroid. This gives a geometric embedding of the dual complex of an Eulerian branching greedoid. It also yields a geometric proof for the root-independence of the greedoid polynomial of an Eulerian digraph.

[1] Kálmán, Tamás and Lilla Tóthmérész, *h^*-vectors of graph polytopes using activities of dissecting spanning trees*, arXiv:2203.17127, 2022.

[2] Lilla Tóthmérész, *A geometric proof for the root-independence of the greedoid polynomial of Eulerian branching greedoids* , arXiv:2204.12419, 2022.

Cut Complexes

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(This talk is based on joint work with Margaret Bayer, Mark Denker, Rowan Rowlands, Sheila Sundaram, and Lei Xue.)

The talk presents our project which is studying two new classes of simplical complexes constructed from graphs, called *cut complexes* and *total cut complexes*. For a graph $G = (V, E)$, a set $S \subset V$ is a *separating set of size* $|S|$, if the induced subgraph $G[V \setminus S]$ (on the vertex set $V \setminus S$) is disconnected. Define the following simplicial complexes associated with a graph G and an integer $k \geq 2$:

• *k-cut complex* $\Delta_k(G)$: the simplicial complex whose facets are the separating sets of G of size $(n-k)$;

• *total k-cut complex* $\Delta_k^t(G)$: the simplicial complex whose facets are the separating sets of size $(n-k)$, with an additional property that their complements are independent sets.

A motivation for this project comes from a result which shows an interesting path leading from graph theory, through squarefree monomial ideals, and then (by using Stanley-Reisner theory) to the combinatorial structure of simplicial complexes, as presented in the following theorem.

Theorem 1 (Fröberg 1990 [2], Eagon and Reiner 1996 [1])**.** *$\Delta_2(G)$ is shellable if and only if G is chordal (no induced cycle of size greater than 3).*

We present some results about the combinatorics and topology of complexes $\Delta_2(G)$, and various results about the structure of $\Delta_k(G)$ for $k \geq 3$. For example, we give some sufficient conditions on graphs such that their k-cut complexes are shellable, and show the effects of common graph operations (disjoint union, join and wedge product) on the shellability of cut complexes. Also, we present combinatorial properties and homotopy types for the cut complexes of the most important classes of graphs (complete bipartite graphs, cycles, forests, grid graphs, etc.). At the end of the talk, we will briefly mention some similar results for total cut complexes $\Delta_k^t(G)$.

[1] John A. Eagon and Victor Reiner. Resolutions of Stanley-Reisner rings and Alexander duality. *J. Pure Appl. Algebra*, 130(3):265–275, 1998.

[2] Ralf Fröberg. On Stanley-Reisner rings. In *Topics in algebra, Part 2* (Warsaw, 1988), volume 26 of *Banach Center Publ.*, pages 57–70. PWN, Warsaw, 1990.

Erdős-Ko-Rado for flags in spherical buildings

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(This talk is based on joint work with Jan De Beule and Klaus Metsch.)

Over the last few years, Erdős-Ko-Rado theorems have been found in many different geometrical contexts including for example sets of subspaces in projective [2] or polar spaces [3]. A recurring theme throughout these theorems is that one can find sharp upper bounds by applying the Delsarte-Hoffman coclique bound to a matrix belonging to the relevant association scheme. In the aforementioned cases, the association schemes turn out to be commutative, greatly simplifying the matter. However, when we do not consider subspaces of a certain dimension but more general flags, we lose this property. In this talk, we will explain how to overcome this problem, using a result originally due to Brouwer [1]. This result, which has seemingly been flying under the radar so far, allows us to derive upper bounds for certain flags in projective spaces and general flags in polar spaces and exceptional geometries. We will show how Chevalley groups, buildings, Iwahori-Hecke algebras and representation theory tie into this story and discuss their connections to the theory of non-commutative association schemes.

[1] Andries Brouwer. The eigenvalues of oppositeness graphs in buildings of spherical type. *Combinatorics And Graphs*. **531** pp. 1-10 (2010).

[2] Chris Godsil & Karen Meagher. Erdős-Ko-Rado theorems: algebraic approaches. *Cambridge University Press*, Cambridge (2016).

[3] Valentina Pepe, Leo Storme & Frédéric Vanhove. Theorems of Erdős-Ko-Rado type in polar spaces. *J. Combin. Theory Ser. A*. **118**, 1291-1312 (2011).

Wednesday 10:30, George Fox Lecture Theatre 1

Embedding problems in sparse expanders

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(This talk is based on joint work with Rajko Nenadov and Michael Krivelevich.)

We develop a general embedding method based on the Friedman-Pippenger tree embedding technique and its algorithmic version, enhanced with a roll-back idea allowing a sequential retracing of previously performed embedding steps. We use this method to obtain the following results.

- We show that the size-Ramsey number of logarithmically long subdivisions of bounded degree graphs is linear in their number of vertices, settling a conjecture of Pak (2002).
- We give a deterministic, polynomial time online algorithm for finding vertex-disjoint paths of a prescribed length between given pairs of vertices in an expander graph. Our result answers a question of Alon and Capalbo (2007).
- We show that relatively weak bounds on the spectral ratio λ/d of d-regular graphs force the existence of a topological minor of K_t where $t = (1 - o(1))d$. We also exhibit a construction which shows that the theoretical maximum $t = d + 1$ cannot be attained even if $\lambda = O(\sqrt{d})$. This answers a question of Fountoulakis, Kühn and Osthus (2009).

On an extremal problem for multigraphs

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(This talk is based on joint work with A. Nicholas Day, Vojtěch Dvořák, Adva Mond, Andrew Treglown and Victor Souza.)

An (n, s, q)-graph is an n-vertex multigraph in which every s-set of vertices supports at most q edges, counting multiplicities. The Turán-type problem of determining how large the sum of the edge multiplicities in an (n, s, q)-graph can be has been studied since the 1990s, and was asymptotically resolved by Füredi and Kündgen [3].

More recently, Mubayi and Terry [4, 5] posed the problem of determining the maximum possible value of the *product* of the edge multiplicities in an (n, s, q)-graph, with motivation coming from applications of container theory. Product-maximisation in these locally sparse multigraphs has a rather different flavour, and some exotic features such as extremal constructions in which parts contain a transcendental proportion of the vertices.

In this talk I will survey what is known about the Mubayi–Terry problem and present some recent progress in the area.

[1] A. Nicholas Day, Victor Falgas-Ravry and Andrew Treglown, Extremal problems for multigraphs, *Journal of Combinatorial Theory Series B* **154** (2022), 1–48.

[2] Victor Falgas-Ravry, On an extremal problem for locally sparse multigraphs, preprint (2021), arXiv:2101.03056.

[3] Zoltán Füredi and André Kündgen, Turán problems for integer-weighted graphs, *Journal of Graph Theory* **40**(4) (2002), 195–225.

[4] Dhruv Mubayi and Caroline Terry, Extremal theory of locally sparse multigraphs, *SIAM Journal on Discrete Mathematics* **34**(3) (2020), 1922–1943.

[5] Dhruv Mubayi, Dhruv and Caroline Terry, An extremal graph problem with a transcendental solution, *Combinatorics, Probability & Computing* **28**(2) (2019), 303–324.

On the Anti-Ramsey Threshold for non-balanced graphs

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(This talk is based on joint work with Taísa Martins, Letícia Mattos, Walner Mendonça, Luiz Moreira, Guilherme O. Mota.)

For graphs G, H, we write $G \xrightarrow{\text{rb}} H$ if any proper edge-coloring of G contains a rainbow copy of H, i.e., a copy where no color appears more than once. Kohayakawa, Konstadinidis and the last author proved that the threshold for $G(n, p) \xrightarrow{\text{rb}} H$ is at most $n^{-1/m_2(H)}$. Previous results have matched the lower bound for this anti-Ramsey threshold for cycles and complete graphs with at least 5 vertices.

Kohayakawa, Konstadinidis and the last author also presented an infinite family of graphs H for which the anti-Ramsey threshold is asymptotically smaller than $n^{-1/m_2(H)}$. In this paper, we devise a framework that provides a richer and more complex family of such graphs that includes all the previously known examples.

Canonical Graph Decompositions via Coverings

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(This talk is based on joint work with Reinhard Diestel, Raphael W. Jacobs, Paul Knappe.)

We present a canonical way to decompose finite graphs into highly connected local parts. The decomposition depends only on an integer parameter whose choice sets the intended degree of locality. The global structure of the graph, as determined by the relative position of these parts, is described by a coarser *model*. This is a simpler graph determined entirely by the decomposition, not imposed.

The model and decomposition are obtained as projections of the tangle-tree structure of a covering of the given graph that reflects its local structure while unfolding its global structure. In this way, the tangle theory from graph minors is brought to bear canonically on arbitrary graphs, which need not be tree-like.

Our theorem extends to locally finite quasi-transitive graphs, and in particular to locally finite Cayley graphs. It thereby offers a canonical decomposition for finitely generated groups into local parts, whose relative structure is displayed by a graph.

[1] J. Carmesin, *Local 2-separators*, JCTB 2022.

[2] R. Diestel, R.W. Jacobs, P. Knappe, J. Kurkofka, *Canonical Graph Decompositions via Coverings*, in preparation.

Some progress on Woodall's Conjecture on packing dijoins in digraphs

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(This talk is based on joint work with Gérard Cornuéjols and Michael Zlatin.)

Let $D = (V, A)$ be a digraph. A *dicut* is the set of arcs in a cut where all the arcs cross in the same direction, and a *dijoin* is a set of arcs whose contraction makes D strongly connected. It is known that every dicut and dijoin intersect. Suppose every dicut has size at least k. *Woodall's Conjecture*, an important open question in Combinatorial Optimization, states that there exist k pairwise disjoint dijoins. We make a step towards resolving this conjecture by proving that A can be decomposed into two sets A_1 and A_2, where A_1 is a dijoin, and A_2 intersects every dicut in at least $k-1$ arcs. We prove this by a *Decompose, Lift, and Reduce (DLR) procedure*, in which D is turned into a *sink-regular* $(k, k+1)$*-bipartite digraph*. From there, by an application of Matroid Optimization tools, we prove the result.

The DLR procedure works more generally for weighted digraphs, and exposes an intriguing number-theoretic aspect of Woodall's Conjecture. In fact, under natural number-theoretic conditions, Woodall's Conjecture and a weighted extension of it are true. By pushing the barrier here, we expose strong base orderability as a key notion for tackling Woodall's Conjecture.

Hamilton Cycles on Dense Regular Digaphs and Oriented Graphs

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(This talk is based on joint work with Allan Lo and Viresh Patel.)

A (directed) cycle in a (directed) graph traversing all the vertices exactly once is called a Hamilton cycle. We prove that for every $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon)$ such that every regular oriented graph on $n > n_0$ vertices and degree at least $(1/4 + \varepsilon)n$ has a Hamilton cycle. This establishes an approximate version of a conjecture of Jackson from 1981. We also establish a result related to a conjecture of Kühn and Osthus about the Hamiltonicity of regular directed graphs with suitable degree and connectivity conditions.

On Diameter and Size in Graphs and Digraphs

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(This talk is based on joint work with Peter Dankelmann.)

In a connected, finite graph or a strong, finite digraph G of order n, the distance $d_G(u, v)$ between two vertices u and v is the length of a shortest $u - v$ path in G. The diameter $diam(G)$ of G is the largest of the distances between all pairs. The (vertex)-connectivity $\kappa(G)$ and edge-connectivity $\lambda(G)$ of G are the minimum number of vertices and edges, respectively, whose removal results in a graph that is not connected or a digraph that is not strong.
Bounds on diameter in terms of order, size and vertex-connectivity were given by Ore in 1968 for graphs and the extension to strong digraphs by Dankelmann in 2021. In the late 80's Caccettta and Smyth strengthened these bounds for edge- connectivity $\lambda \geq 8$ instead of vertex-connectivity. Sharp bounds on the diameter for the remaining values of λ, i.e, for $2 \leq \lambda \leq 7$ were given by Dankelmann in 2021 who also extended these results to Eulerian digraphs.
In this talk, we present these existing results and the extension to the results of Caccetta and Smyth, and Dankelmann to new results for bipartite graphs with close consideration of results presented by Mukwembi on order, size, diameter and minimum degree in 2013. Finally we will discuss also the extension of these new results for bipartite graphs to Eulerian bipartite digraphs.

Wednesday 11:45, George Fox Lecture Theatre 2

Small and Disjoint Quasi-Kernels

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(This talk is based on joint work with Jiangdong Ai, Stefanie Gerke and Gregory Gutin.)

A *quasi-kernel* of a directed graph D is an independent set $Q \subseteq V(D)$ such that for every vertex $v \in V(D) \backslash Q$, there exists a directed path with one or two arcs from v to a vertex $u \in Q$. In 1976, Erdős and Śzekely conjectured that every sink-free digraph $D = (V, A)$ has a quasi-kernel of size at most $|V|/2$. In this paper, we prove a slightly stronger result which implies that the conjecture holds for the anti-claw-free digraphs. In addition, we show that this conjecture holds for sink-free digraphs with a quasi-kernel Q that satisfies that for all $u \in Q$, $N^+(u) \cap N^-(u) \neq \emptyset$. For sink-free kernel-perfect, critical kernel-imperfect, unicyclic, and a class of semicomplete compositions, we show a stronger result. Namely, graphs belonging to these classes have a pair of disjoint quasi-kernels.

Optimal Resistor Networks

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(This talk is based on joint work with Mark Walters.)

A graph can be regarded as an electrical network by replacing each edge with a 1 ohm resistor. This viewpoint has applications to some diverse areas of mathematics including random walks, partitioning rectangles into squares, and statistical design theory.

A statistical application motivates our main problem. Given a graph on n vertices with m edges, how small can the average resistance between pairs of vertices be?

There are two very plausible extremal constructions – graphs like a star, and graphs which are close to regular – with the transition between them occuring when the average degree is 3. However, surprisingly, there are significantly better constructions for a range of average degree including average degree near 3. We will discuss this behaviour and other results and open questions related to this problem.

Tight Hamilton cycles in uniformly dense k-uniform hypergraphs

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(This talk is based on joint work with Pedro Araújo and Mathias Schacht.)

We study tight Hamilton cycles in quasirandom hypergraphs with minimum degree at least $\Omega(n^{k-1})$. For 3-uniform hypergraphs and different notions of quasirandomness these type of problems were studied previously by Aigner-Horev and Levy, Gan and Han, and the authors. We generalise those results for k-uniform hypergraphs.

For one notion of quasirandomness and under a minimum degree condition of $\Omega(n^{k-1})$, we obtain an asymptotically optimal density threshold that enforces the existence of a tight Hamilton cycle. Moreover, we prove that under the same minimum degree conditions, for stronger notions of quasirandomness, any arbitrarily small density is already enough to ensure the existence of such a cycle. Additionally, for weaker notions, we provide examples of k-uniform hypergraphs with quasirandom density almost 1 and subject to the same minimum degree condition, that do not contain tight Hamilton cycles.

Permutation limits at infinitely many scales

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In this talk we will investigate convergence of sequences of permutations at different scales. Let S_n denote the set of permutations of length n. An *occurrence* of pattern $\pi \in S_k$ in permutation $\sigma \in S_n$ (with $k \leqslant n$) is a k-element subset of indices $1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n$ whose image $\sigma(i_1) \ldots \sigma(i_k)$ under σ is order-isomorphic to π. We say that such an occurrence has *width* $i_k - i_1 + 1$. Given a real number $f \in [k, n]$, let $\nu_f(\pi, \sigma)$ be the number of occurrences of π in σ having width no greater than f. Then the density of π in σ *at scale* f, denoted $\rho_f(\pi, \sigma)$, is $\nu_f(\pi, \sigma) / \binom{n}{k}_f$, where $\binom{n}{k}_f$ is the number of k-element subsets of $[n]$ of width at most f.

Given a *scaling function* $f = f(n) \gg 1$, an infinite sequence $(\sigma_j)_{j \in \mathbb{N}}$ of permutations with $|\sigma_j| \to \infty$ is *convergent at scale* f if $\rho_f(\pi, \sigma_j) = \rho_{f(|\sigma_j|)}(\pi, \sigma_j)$ converges for every pattern π. That is, there exists an infinite vector $\Xi \in [0,1]^S$ (where S is the set of all permutations), which we call a *scale limit*, such that $\rho_f(\pi, \sigma_j) \to \Xi_\pi$ for all $\pi \in S$. The set of possible scale limits does not depend on the scale:

Theorem 1. *If Ξ is any scale limit and $f \ll n$, then there exists a sequence of permutations convergent to Ξ at scale f.*

If $f \ll g$, then convergence at scale f is independent of convergence at scale g:

Theorem 2. *Let $\{f_t : t \in \mathbb{N}\}$ be any countably infinite set of scaling functions totally ordered by $\ll$, and for each $t \in \mathbb{N}$, let Ξ_t be any scale limit. Then there exists a sequence of permutations which converges to Ξ_t at scale f_t for each $t \in \mathbb{N}$.*

In the case of *global* convergence, when $f = n$, one can represent the limit by a *permuton*, a probability measure on the unit square with uniform marginals. What can we say when $f \ll n$? A scale limit cannot always be represented by a permuton. However, it is believed that certain probability distributions over permutons (that is, *random permutons*) suffice:

Question 3. *Can every scale limit be represented by a random permuton? If so, which random permutons are scale limits?*

A sequence of permutations is *scalably convergent* if it converges to the same limit (a *scalable limit*) at every scale $f \ll n$. A permuton is *tiered* if it can be partitioned into a countable number of horizontal *tiers* $[0,1] \times [a,b]$ such that in each tier the mass is uniformly distributed either on the whole tier or else along one of its diagonals. It seems likely that tiered permutons are sufficient to characterise scalable limits:

Question 4. *Can every scalable limit be represented by a random tiered permuton? If so, which random tiered permutons are scalable limits?*

[1] David Bevan. Independence of permutation limits at infinitely many scales. *JCTA* 186:105557, 2022.

EMBEDDING $K_{3,3}$ AND K_5 ON ORIENTABLE SURFACES

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(This talk is based on joint work with William L. Kocay, University of Manitoba, Winnipeg, Canada)

The Kuratowski graphs $K_{3,3}$ and K_5 are well-known fundamental non-planar graphs that characterize planarity. We are interested in obtaining all their distinct 2-cell embeddings on orientable surfaces. Counting distinct 2-cell embeddings of these two graphs on orientable surfaces was previously done by Mull [1] and Mull et al. [2], using Burnside's Lemma and automorphism groups of the graphs, without actually constructing the embeddings. The 2-cell embeddings of $K_{3,3}$ and K_5 on the torus are well-known. We obtain all 2-cell embeddings of $K_{3,3}$ and K_5 on the double torus, using a constructive approach, starting with their common minor Θ_5, which is a multi-graph consisting of two vertices and a set of five parallel edges between them. First, we prove that there are exactly three distinct 2-cell embeddings of Θ_5 on the double torus (see Figure 1). Then, we show that there is a unique non-orientable 2-cell embedding of $K_{3,3}$, and 14 orientable and 17 non-orientable 2-cell embeddings of K_5 on the double torus. These are explicitly obtained by recursively expanding from minors. Therefore we confirm the numbers of embeddings obtained by Mull [1] and Mull et al. [2] for the double torus. As a consequence, several new polygonal representations of the double torus are presented. Using an exhaustive search, rotation systems for the one-face embeddings of K_5 on the triple torus are also found.

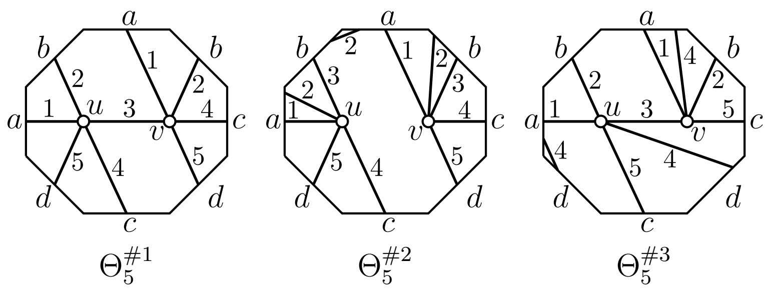

Figure 1: The 2-cell embeddings of Θ_5 on the double torus.

[1] B.P. Mull, Enumerating the orientable 2-cell imbeddings of complete bipartite graphs, *J. Graph Theory* **30** (1999), 77–90.

[2] B.P. Mull, R.G. Rieper, A.T. White, Enumerating 2-cell imbeddings of connected graphs, *Proc. Amer. Math. Soc.* **103**(1) (2008), 321–330.

Braced Triangulations and Rigidity

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This talk is based on joint work with J. Cruickshank, D. Kitson and B. Schulze.

Triangulations of the 2-sphere play an important role in rigidity theory of bar-joint frameworks. Gluck has shown that generic realisations of these graphs as bar-joint frameworks in the 3-dimensional Euclidean space are minimally rigid.

In this talk, we consider triangulated spheres with a fixed number of additional edges (braces). We shall show that for any $b \in \mathbb{N}$ there exists an inductive construction, based on vertex splitting, of triangulations with b braces, having finitely many base graphs. In particular, we establish a bound for the maximum size of a base graph with b braces that is linear in b. For $b = 1$ and $b = 2$ we determine the list of base graphs explicitly.

Applying the above results we show that doubly braced triangulations are (generically) minimally rigid in two distinct geometric contexts arising from a hypercylinder in $\mathbb{R}^4$ and a class of mixed norms on $\mathbb{R}^3$.

Unique Realisations of Outerplanar Graphs

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(This talk is based on joint work with James Cruickhank and Shin-Ichi Tanigawa.)

A *braced outerplanar graph* is any graph which can be obtained by adding extra edges to an outerplanar graph. We show that a convex realisation of a braced maximal outerplanar graph is uniquely defined by its edge lengths if and only if it is 3-connected.

When is a rod configuration infinitesimally rigid?

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(This talk is based on joint work with Klara Stokes and Lars-Daniel Öhman.)

A rod configuration is a realisation of a hypergraph as points and straight lines in the plane, where the lines behave as rigid bodies. Tay and Whiteley conjectured that the infinitesimal rigidity of rod configurations realising 2-regular hypergraphs depends only on the generic rigidity of body-and-joint frameworks realising the same hypergraph [3]. This conjecture is known as the molecular conjecture because of its applications to molecular chemistry.

In 1989, Whiteley proved a version of the molecular conjecture for hypergraphs of arbitrary degree that can be realised as independent body-and-joint frameworks in the plane [4]. In 2008, Jackson and Jordán proved the molecular conjecture in the plane, and Katoh and Tanigawa proved it in arbitrary dimension in 2011 [1, 2].

In this talk, we will see that the infinitesimal rigidity of a sufficiently generic rod configuration realising an arbitrary hypergraph depends only on the generic rigidity of an associated graph, which we call a cone graph. This result can be seen as a generalisation of Whiteley's version of the molecular conjecture to arbitrary hypergraphs.

[1] B. Jackson and T. Jordán. Pin-collinear body-and-pin frameworks and the molecular conjecture. Discrete Comput. Geom. 40:2 (2008) 258–278.

[2] N. Katoh and S. Tanigawa. A proof of the molecular conjecture. Discrete Comput. Geom. 45:4 (2011) 647–700.

[3] T.S. Tay and W. Whiteley. Recent advances in the generic rigidity of structures. Structural Topology. 9 (1984) 31–38.

[4] W. Whiteley. A matroid on hypergraphs, with applications in scene analysis and geometry. Discret. Comput. Geom. An International Journal of Mathematics and Computer Science. 4 (1989) 278–301.

[5] A. Nixon, B. Schulze and W. Whiteley. Rigidity through a Projective Lens. Applied Sciences 11:24 (2021)

Dirac-type results for tilings and coverings in ordered graphs

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(This talk is based on joint work with Andrew Treglown.)

A *(vertex) ordered graph* or *labelled graph* H on h vertices is a graph whose vertices have been labelled with $\{1, \ldots, h\}$. In recent years there has been a significant effort to develop both Turán and Ramsey theories in the setting of vertex ordered graphs (see for example [1, 3, 4, 5]). Motivated by this line of research, Balogh, Li and Treglown [2] recently initiated the study of Dirac-type problems for ordered graphs. In particular, they focused on the problem of determining the minimum degree threshold for forcing a perfect H-tiling in an ordered graph for any fixed ordered graph H (recall that a perfect H-tiling in a graph G is a collection of vertex-disjoint copies of H covering all the vertices in G). In this talk we present a result which builds up on the ideas from [2] and fully resolve such problem. This provides an ordered graph analogue of the seminal tiling theorem of Kühn and Osthus [Combinatorica 2009]. We also determine the asymptotic minimum degree threshold for forcing an H-cover in an ordered graph (for any fixed ordered graph H).

[1] M. Balko, J. Cibulka, K. Král and J. Kynčl, Ramsey numbers of ordered graphs, *Electr. J. Combin.* **27** (2020), P1.16.

[2] J. Balogh, L. Li and A. Treglown, Tilings in vertex ordered graphs, *J. Combin. Theory Ser. B* **155** (2022), 171–201.

[3] D. Conlon, J. Fox, C. Lee and B. Sudakov, Ordered Ramsey numbers, *J. Combin. Theory Ser. B* **122** (2017), 353–383.

[4] J. Pach and G. Tardos, Forbidden paths and cycles in ordered graphs and matrices, *Israel J. Math.* **155** (2006), 359–380.

[5] G. Tardos, Extremal theory of vertex or edge ordered graphs, in Surveys in Combinatorics 2019 (A. Lo, R. Mycroft, G. Perarnau and A. Treglown eds.), London Math. Soc. Lecture Notes 456, 221–236, Cambridge University Press, 2019.

Erdős's conjecture on the pancyclicity of Hamiltonian graphs

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(This talk is based on joint work with Nemanja Draganić and Benny Sudakov.)

An n-vertex graph is *Hamiltonian* if it contains a cycle covering all its vertices and it is *pancyclic* if it contains cycles of all lengths from 3 up to n. In 1973, Bondy stated his celebrated meta-conjecture that any non-trivial condition which implies that a graph is Hamiltonian should also imply that it is pancyclic (up to a certain collection of simple exceptional graphs). As an example, consider the classical Dirac's theorem stating that every n-vertex graph with minimum degree at least $n/2$ is Hamiltonian. Strengthening this, Bondy himself showed that every such graph is in fact either pancyclic or isomorphic to the complete bipartite graph $K_{n/2,n/2}$.

Bondy's meta-conjecture deals with conditions for Hamiltonicity which imply pancyclicity. In a similar fashion, one can ask the following natural question: Let G be a Hamiltonian graph; under which assumptions can we guarantee that G is also pancyclic? Indeed, also in the 1970s, Erdős put forward the problem below.

Question 1. *Given an n-vertex Hamiltonian graph with independence number $\alpha(G) \leq k$, how large does n have to be in terms of k in order to guarantee that G is pancyclic?*

He proved that it is enough to have $n = \Omega(k^4)$ and conjectured that already $n = \Omega(k^2)$ should be enough - a simple construction shows that this is best possible. Since then there have been several improvements of Erdős's initial result – by Keevash and Sudakov who proved that $n = \Omega(k^3)$ is enough, by Lee and Sudakov who improved it to $n = \Omega(k^{7/3})$, and finally by Dankovics who showed that $n = \Omega(k^{11/5})$ suffices. We resolve the conjecture of Erdős, showing that if a Hamiltonian graph G has $n = \Omega(k^2)$ vertices and $\alpha(G) \leq k$, then G is pancyclic.

Copies of oriented trees with many leaves in tournaments

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(This talk is based on joint work with Richard Montgomery.)

Given an n-vertex oriented tree T, how large must a tournament G be, in order to guarantee G contains a copy of T? A strengthening of Sumner's conjecture poses that, if T has k leaves, then it is enough for G to have $(n+k-1)$ vertices. While this conjecture has been recently confirmed in the case where k is fixed and n is allowed to grow large, it remains open for trees with a large proportion of leaves. In this talk, we confirm this conjecture holds approximately, even in the many-leaves case. We also discuss how the techniques behind this approximate result extend to a different setting in which we consider the maximum degree of T, instead of the number of leaves.

Thursday 11:45, George Fox Lecture Theatre 1

Cycle decompositions in k-uniform hypergraphs

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(This talk is based on joint work with Simón Piga and Nicolás Sanhueza-Matamala.)

We show that k-uniform hypergraphs on n vertices whose codegree is at least $(2/3+o(1))n$ can be decomposed into tight cycles, subject to the trivial divisibility condition that every vertex degree is divisible by k. As a corollary, we show that such hypergraphs also have a tight Euler tour answering a question of Glock, Joos, Kühn and Osthus.

Distinct dot products and arithmetic growth

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(This talk is based on joint work with Brandon Hanson and Steven Senger.)

A variant of the Erdős distinct distance problem is to consider the minimum number of dot products determined by a set of N points in the plane. A simple incidence geometric argument proves that there are at least $N^{2/3}$ such dot products. I will discuss joint work with Hanson and Senger which improves this bound.

Thursday 10:55, George Fox Lecture Theatre 2

Partition and density regularity for polynomial systems

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(This talk is based on joint work with Sam Chow.)

A system of polynomial equations is called *partition regular* if every finite colouring of the positive integers produces monochromatic solutions to the system. A system is called *density regular* if it has solutions over every set of integers with positive upper density. A classical theorem of Rado characterises partition regularity for linear systems, whilst Szemerédi's theorem classifies all density regular linear systems. In this talk, I will report on recent developments on the classification of partition and density regularity for sufficiently non-singular systems of polynomial equations.

Equidistribution of high rank boolean polynomials over $\mathbb{F}_p$

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(This talk is partly based on joint work with Timothy Gowers.)

Let $d \geq 2$ be a positive integer. For a polynomial P in several variables over a field $\mathbb{F}$ and with total degree d, we say that the rank $\operatorname{rk} P$ of P is the smallest nonnegative integer k such that there exist polynomials $Q_1, \dots, Q_k, R_1, \dots, R_k$ all with degree at most $d-1$ such that we can write

$$P = Q_1 R_1 + \cdots + Q_k R_k$$

In the case $\mathbb{F} = \mathbb{F}_p$, we will generalise a result of Green and Tao about equidistribution of high rank polynomials to the case where the range of the variables is restricted to an arbitrary subset of $\mathbb{F}$.

Theorem 1. *Let p be a prime integer, let $2 \leq d < p$ be a positive integer and let S be an arbitrary non-empty subset of $\mathbb{F}_p$. There exists a function $A_{p,d} : \mathbb{R}_+ \to \mathbb{R}_+$ such that for every $\epsilon > 0$, if P is a polynomial over $\mathbb{F}_p$ with $\deg P = d$ and such that there exists $t \in \mathbb{F}_p^*$ satisfying*

$$|\mathbb{E}_{x \in S^n} \exp(\tfrac{2\pi i}{p} t P(x))| \geq \epsilon$$

then there exists a polynomial P_0 identically constant on S^n such that $\operatorname{rk}(P - P_0) \leq A_{p,d}(\epsilon)$.

To prove this result we will use two black boxes: the equivalence between the partition rank pr and the analytic rank for tensors, as well as the following result of the author.

Proposition 2. *Let $d \geq 2$, $n \geq 1$ be positive integers and let E be the set of $(x_1, \dots, x_d) \in [n]^d$ such that there exist distinct $i, j \in [d]$ with $x_i = x_j$. There exists a function $G_d : \mathbb{N} \to \mathbb{N}$ such that if $l \geq 1$ is a positive integer and $T : [n]^d \to \mathbb{F}$ is an order d tensor (over an arbitrary field $\mathbb{F}$) such that*

$$\operatorname{pr} T(X_1 \times \cdots \times X_d) \leq l$$

is satisfied for all d-tuples $(X_1, \dots, X_d)$ of pairwise disjoint subsets of $[n]$, then we can find an order d tensor V supported inside E such that

$$\operatorname{pr}(T + V) \leq G_d(l)$$

Thursday 11:45, George Fox Lecture Theatre 2

Bounds on the estimation error of syndrome-based channel parameter estimation by linear codes

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(This talk is based on joint work with Yu Tsunoda.)

It is known that a well-designed binary linear code allows for efficiently and accurately estimating the cross-over probability of the binary symmetric channel by simply looking at the syndrome weight before even attempting error correction. However, while various promising simulation results and heuristic analyses have been provided in the literature, as far as the authors are aware, there are no rigorous arguments for why it is so accurate. Here, for given $0 < \delta < 1$, we prove a tail bound on the probability $\Pr(|\hat{p} - p| \geq \delta p)$ that the estimation $\hat{p}$ of the cross-over probability p by the syndrome weight deviates from the true value by at least δp. When a regular low-density parity-check code is used for estimation, our bound shows that $\Pr(|\hat{p} - p| \geq \delta p)$ tends to 0 exponentially fast as the code length tends to infinity, giving a mathematical explanation of why the estimation method works well. The proof is combinatorial and relies on McDiarmid's inequality.

More on subsystems of Netto triple systems

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(This talk is based on joint work with Darryn E. Bryant and Barbara M. Maenhaut.)

The Netto triple systems are a class of Steiner triple systems having order $q = p^n$ where $n \geq 1$, p is prime, and $q \equiv 7 \pmod{12}$, and there is a unique (up to isomorphism) Netto triple system for each such order. For $q \neq 7$, their full automorphism group acts transitively on unordered pairs of points but not on ordered pairs of points, and they are the only Steiner triple systems with this property.

Netto triple systems are block-transitive, cyclic, uniform, anti-mitre, and are block-regular if and only if $q \equiv 7$ or $31 \pmod{36}$. The elements of a field of order q form the point set of a Netto triple system of order q, and the blocks can be generated from the triple $\{0, 1, \alpha\}$ where α is a primitive sixth root of unity.

We confirm Robinson's 1975 conjecture that prime order Netto triple systems have no non-trivial subsystems, prove that cubic Netto triple systems have only expected subsystems and investigate when Netto triple systems have subsystems other than the expected ones.

Thursday 10:55, George Fox Lecture Theatre 3

An introduction to DPDFs and EPDFs

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(This talk is based on joint work with Sophie Huczynska.)

A Disjoint Difference Family (DDF) is a combinatorial structure formed from a collection of disjoint subsets of a group G, in which each group element occurs precisely lambda times as a difference between two elements of the same subset. An External Difference Family (EDF) is similarly formed by disjoint subsets of G, with each element of G occurring exactly lambda times as a difference between elements of disjoint subsets. Both combinatorial structures have been widely studied and have applications to cryptography.

We call a DDF comprising of just one subset a Difference Set. Difference Sets have a well-studied partial analogue; namely a Partial Difference Set (PDS). In spite of the fact that we can consider a Difference Set to be a restricted type of DDF, the partial analogues of DDFs have not previously been classified. EDFs are again a similar structure with no partial analogue. In this talk, I will introduce a partial analogue for both of these structures and I will set up a cyclotomic framework which may be used to find examples of these structures.

Thursday 11:20, George Fox Lecture Theatre 3

Coefficientwise total positivity of some combinatorial matrices

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Lancaster University

(This talk is based on joint work with X. Chen, B. Deb, A. Dyachenko, A. D. Sokal.)

A (finite or infinite) matrix with real entries is *totally positive* if all of its minors are nonnegative. If we equip the polynomial ring $\mathbb{R}[\mathbf{x}]$ (where $\mathbf{x} = \{x_i\}_{i\geq 0}$ is a set of algebraic indeterminates) with the *coefficientwise partial order* (that is, we say $P \in \mathbb{R}[\mathbf{x}]$ is *nonnegative* if and only if P is a polynomial with nonnegative coefficients), then a matrix with entries belonging to $\mathbb{R}[\mathbf{x}]$ is *coefficientwise totally positive* if all of its minors are polynomials with *nonnegative coefficients*.

In this talk I will present some conjectures and results concerning the matrix

$$T(a, c, d, e, f, g) = (T_{n,k})_{n,k\geq 0}$$

with entries that satisfy a three-term linear recurrence:

$$T_{n,k} = (a(n-k)+c)T_{n-1,k-1} + (dk+e)T_{n-1,k} + (f(n-2)+g)T_{n-2,k-1}$$

for $n \geq 1$ with initial conditions $T_{0,k} = \delta_{k0}$ and $T_{-1,k} = 0$.

Under certain specialisations the entries of $T(a, c, d, e, f, g)$ count a variety of natural combinatorial objects with respect to different statistics. On the other, this matrix appears, and in some cases can be shown to be, coefficientwise totally positive. I will discuss how classical combinatorial techniques can be employed to prove such total positivity results.

[1] X. Chen, B. Deb, A. Dyachenko, T. Gilmore, and A. D. Sokal, *Coefficientwise total positivity of some matrices defined by linear recurrences*, Séminaire Lotharingien de Combinatoire 85B (2021), Proceedings of the 33rd International Conference on Formal Power Series and Algebraic Combinatorics.

Quotient graphs of symmetrically rigid frameworks

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(This talk is based on joint work with Georg Grasegger, Eleftherios Kastis and Anthony Nixon.)

A natural problem in combinatorial rigidity theory concerns the determination of the rigidity or flexibility of bar-joint frameworks in $\mathbb{R}^d$ that admit some non-trivial symmetry. When $d = 2$ there is a large literature on this topic. In particular, it is typical to quotient the symmetric graph by the group and analyse the rigidity of symmetric, but otherwise generic frameworks, using the combinatorial structure of the appropriate group-labelled quotient graph. However, mirroring the situation for generic rigidity, little is known combinatorially when $d \geq 3$. Nevertheless in the periodic case, a key result of Borcea and Streinu [1] characterises when a quotient graph can be lifted to a rigid periodic framework in $\mathbb{R}^d$. We develop an analogous theory for symmetric frameworks in $\mathbb{R}^d$. The results obtained apply to all finite and infinite 2-dimensional point groups, and then in arbitrary dimension they concern a wide range of infinite point groups, sufficiently large finite groups and groups containing translations and rotations. For the case of finite groups we also derive results concerning the probability of assigning group labels to a quotient graph so that the resulting lift is symmetrically rigid in $\mathbb{R}^d$.

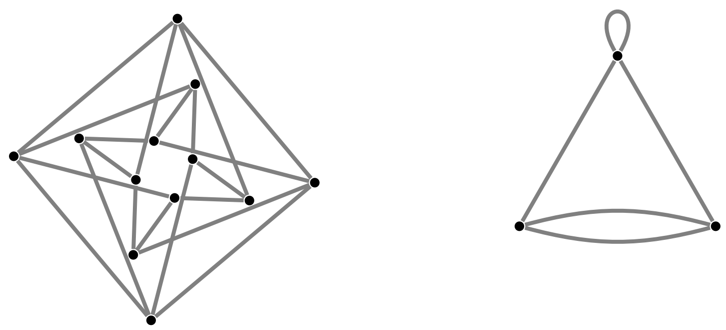

Figure 1: A graph with 4-fold rotational symmetry (left) and its quotient graph (right).

[1] C. S. Borcea, I. Streinu, *Minimally rigid periodic graphs*, Bulletin of the London Mathematical Society 43(6), 2011 pp. 1093–1103. doi: 10.1112/blms/bdr044.

Rigidity of Symmetric Frameworks on the Cylinder

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(This talk is based on joint work with Anthony Nixon, Bernd Schulze.)

A bar-joint framework (G, p) is the combination of a finite simple graph $G = (V, E)$ and a placement $p : V \to \mathbb{R}^d$. The framework is rigid if the only edge-length preserving continuous deformations of the vertices arise from isometries of the space. This talk combines two recent extensions of the generic theory of rigid and flexible graphs by considering symmetric frameworks in $\mathbb{R}^3$ restricted to move on a surface. We give the necessary combinatorial conditions for a symmetric framework on the cylinder to be isostatic (i.e. minimally infinitesimally rigid) under any finite point group symmetry. In two of the 5 possible cases, half turn and inversion symmetry, these conditions are then shown to be sufficient under suitable genericity assumptions, and precise combinatorial descriptions of symmetric isostatic graphs in these contexts are given.

Global Area Rigidity of Generic Hypergraph Frameworks

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(This talk is based on joint work with Louis Theran (academic supervisor).)

Connelly and Gortler, Healy and Thurston showed that global Euclidean rigidity of graph frameworks is a generic property, ie. either all generic frameworks of a graph are globally rigid or none are. In this talk we cover the basics of area rigidity, highlighting its similarities and differences with Euclidean rigidity. We then use the family of hypergraphs defined by triangulations of the 2-sphere to demonstrate that global area rigidity is not a generic property.

Thursday 11:45, George Fox Lecture Theatre 5

Fast algorithms for global rigidity

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(This talk is based on joint work with András Mihálykó.)

A (bar-joint) framework (a collection of rigid bars in $\mathbb{R}^d$ connected by joints that allow full spherical motion) is rigid in $\mathbb{R}^d$, if it cannot be deformed continuously into another non-isomorphic framework. It is globally rigid if no non-isomorphic framework can be given with the same bar lengths. In certain cases (for example, for generic frameworks in the plane or on the cylinder), both the rigidity and the global rigidity of a framework depend only on the underlying graph. In these cases rigidity is often characterized by some sparsity properties of the underlying graph, and global rigidity is characterized by redundant rigidity (where the graph remains rigid after deleting an arbitrary edge) and 2- or 3-vertex-connectivity.

In this talk we first show how the global rigidity of a graph $G = (V, E)$ can be checked in $O(|V|^2)$ time by showing how the above mentioned combinatorial properties can be checked efficiently. As it is known that the 2- or 3-connectivity of a graph can be checked in linear time, the main aim of this algorithm is the testing of redundant rigidity in $O(|V|^2)$ time. We consider this problem on a more general structure, called the (k, ℓ)-sparsity matroid that encapsulates rigidity for several spaces. We also show how the components of the sparsity matroid of a graph $G = (V, E)$ can be calculated in $O(|V|^2)$ time.

The combinatorial characterizations of global rigidity allow us to consider the following as a combinatorial problem (*global rigidity augmentation problem*): given a rigid graph $G = (V, E)$, find a minimum size edge set F so that $G + F$ is globally rigid.

In the second part of this talk, we sketch an $O(|V|^2)$ algorithm to solve the global rigidity augmentation problem and its extension for (k, ℓ)-sparsity matroids. The algorithm uses the above mentioned algorithm which calculates the components of the (k, ℓ)-sparsity matroid as a subroutine. Besides this, it uses the algorithm for redundant rigidity augmentations of minimally rigid (hyper)graphs from [2], the (mostly algorithmic) proof of the min-max theorem given in [1] for the global rigidity augmentation problem, and efficient algorithms which calculates the structures of the 2- or 3-connected blocks of a graph.

[1] Cs. Király and A. Mihálykó. Globally rigid augmentation of rigid graphs. Technical Report TR-2021-04, Egerváry Research Group, Budapest, 2021. egres.elte.hu. To appear in *SIAM J. Disc. Math.*

[2] Cs. Király and A. Mihálykó. Sparse graphs and an augmentation problem. *Math. Program.*, 192(1):119–148, 2022.

Friday 11:25, George Fox Lecture Theatre 1

Graphs with large minimum degree and no small odd cycles are three-colourable

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(This talk is based on joint work with Julia Böttcher, Domenico Mergoni, Olaf Parczyk and Jozef Skokan.)

Let $\mathcal{F}$ be a fixed family of graphs. The *homomorphism threshold* of $\mathcal{F}$ is the infimum of those α for which there exists an $\mathcal{F}$-free graph $H(\mathcal{F}, \alpha)$, such that every $\mathcal{F}$-free graph on n vertices of minimum degree αn is homomorphic to $H(\mathcal{F}, \alpha)$. Letzter and Snyder showed that the homomorphism threshold of $\{C_3, C_5\}$ is $1/5$. They found explicit graphs $H(\mathcal{F}, \alpha)$ for $\alpha \geq \frac{1}{5} + \varepsilon$, which were in addition 3-colourable. Thus, their result also implies that $\{C_3, C_5\}$-free graphs of minimum degree at least $(\frac{1}{5} + \varepsilon)n$ are 3-colourable. For longer cycles, Ebsen and Schacht showed that the homomorphism threshold of $\{C_3, C_5, \ldots, C_{2\ell-1}\}$ is $\frac{1}{2\ell-1}$. However, their proof does not imply a good bound on the chromatic number of $\{C_3, \ldots, C_{2\ell-1}\}$-free graphs of minimum degree $(\frac{1}{2\ell-1} + \varepsilon)n$. Answering a question of Letzter and Snyder, we prove that such graphs are 3-colourable.

Unavoidable patterns in 2-edge colorings of the complete bipartite graph

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(This talk is based on joint work with Dr. Adriana Hansberg.)

Ramsey's theorem states that, given a graph G and a large enough integer n, any coloring of the edges of K_n contains a monochromatic copy of G. Typical Ramsey results guarantee the existence of monochromatic substructures. However, the search for non-monochromatic substructures is interesting as well. It is known that any 2-coloring of the edges of a large enough complete graph with enough edges in each color contains at least one of two patterns, either a colored K_{2t} where one color class induces a K_t or a colored K_{2t} where one color class induces two disjoint K_t's. This result has given rise to many interesting problems involving balanceability (which seeks structures with equal proportions of color) and omnitonality (which seeks structures with all possible proportions of color). In this talk, we will discuss the unavoidable patterns found when we color the edges of a large enough complete bipartite graph with two colors and their significance on the search of balanceable and omnitonal structures.

Subgraph densities in K_r-free graphs

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(This talk is based on joint work with Andrzej Grzesik, Nika Salia and Casey Tompkins.)

For graphs H and F, the generalized Turán number $ex(n, H, F)$ is defined to be the maximum number of (not necessarily induced) copies of H in an n-vertex graph G which does not contain F as a subgraph. Estimating $ex(n, H, F)$ for various pairs H and F has been a central topic of research in extremal combinatorics. The case when H and F are both cliques was settled early on by Zykov and independently by Erdős. The problem of maximizing 5-cycles in a triangle-free graph was a long-standing open problem. The problem was finally settled by Grzesik and independently by Hatami, Hladký, Král, Norine and Razborov. In the case when the forbidden graph F is a triangle and H is any bipartite graph containing a matching on all but at most one of its vertices, $ex(n, H, F)$ was determined exactly by Győri, Pach and Simonovits in 1991. More recently there has been extensive work on the topic following the work of Alon and Shikhelman, who introduced the extremal function $ex(n, H, F)$ for general pairs H and F.

For a given n and a double star $S_{a,b}$, Győri, Wang and Woolfson proved that there exists n' such that for all triangle-free graphs G on n the number of copies of $S_{a,b}$ in G is at most the number of copies of it in $K_{n',n-n'}$ plus an error term $o(n^{a+b+2})$.

Recently Lidický and Murphy proposed the following natural conjecture.

Conjecture 1 (Lidický, Murphy). *Let H be a graph and let r be an integer such that $r > \chi(H)$. Then there exist integers $n_1, n_2, \ldots, n_{r-1}$ such that $n_1 + n_2 + \cdots + n_{r-1} = n$ and we have*

$$ex(n, H, K_r) = H(K_{n_1,n_2,\ldots,n_{r-1}}).$$

Unfortunately, the conjecture is not true in general. We present some counterexamples in the talk. However, it is natural to consider the following modification of Conjecture 1.

Conjecture 2. *Let G be a graph with diameter at most $2r - 2$ with $\chi(G) < r$, then $ex(n, G, K_r)$ is asymptotically achieved by a blow-up of K_{r-1}.*

As a first step towards Conjecture 2 for $r = 3$, we proved it for all bipartite graphs of radius 2 and some other bipartite graphs.

Friday 14:15, George Fox Lecture Theatre 1

Edge Contraction and Forbidden Induced Graphs

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A graph G is H-free if any subset of $V(G)$ does not induce a subgraph of G that is isomorphic to H. Given a graph H, we present sufficient and necessary conditions for a graph G such that G/e is H-free for any edge e in $E(G)$. Afterwards, we use these conditions to characterize forests, claw-free, $2K_2$-free, C_4-free, C_5-free, and split graphs.

Maximising Minimum Reachability in Temporal Graphs

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Temporal graphs consist of an underlying graph (G, E) and an assignment t of timesteps to edges that specifies when each edge is active. This allows us to model spread through a network which is time-sensitive. We will consider a fixed set of timesteps for a given graph and reorder them to optimise reachability. Previous work has mainly explored minimising spread for applications such as epidemiology. Here, we will be looking at the opposite problem of increasing movement through a graph. Maximising spread can be useful in situations where we would like information or resources to be shared efficiently, such as advertising or even vaccine rollout.

In particular, our goal is to reorder the timesteps assigned to the edges in our graph such that the minimum number of vertices reachable from any starting vertex is maximised. We will discuss optimal ordering in specific graphs and features of more general graphs which allow for high minimum reachability.

Classifying Subset Feedback Vertex Set for H-Free Graphs

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Daniël Paulusma, Paweł Rzążewski

In the Feedback Vertex Set problem, we aim to find a small set S of vertices in a graph intersecting every cycle. The Subset Feedback Vertex Set problem requires S to intersect only those cycles that include a vertex of some specified set T. We also consider the Weighted Subset Feedback Vertex Set problem, where each vertex u has weight $w(u) > 0$ and we ask that S has small weight. By combining known NP-hardness results with new polynomial-time results we prove full complexity dichotomies for Subset Feedback Vertex Set and Weighted Subset Feedback Vertex Set for H-free graphs, that is, graphs that do not contain a graph H as an induced subgraph.

[1] G. Paesani, Daniël Paulusma and Paweł Rzążewski, Classifying Subset Feedback Vertex Set for H-Free Graphs, Proc. WG 2022, Lecture Notes in Computer Science, to appear.

A random Hall-Paige conjecture

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(This talk is based on joint work with Alexey Pokrovskiy.)

A complete mapping of a group G is a bijection $\phi\colon G \to G$ such that $x \mapsto x\phi(x)$ is also bijective. The Hall-Paige conjecture from 1955 states that G has a complete mapping whenever the product of all elements of G is contained in the commutator subgroup of G. The conjecture is a theorem since 2009 thanks to breakthrough work of Wilcox, Evans, and Bray.

We will discuss a generalisation of the Hall-Paige conjecture for random subsets of groups. The resulting statement applies only to large groups, but is flexible enough to address many longstanding problems in combinatorial group theory. A sample application is a characterisation of (large) groups whose elements can be ordered so that the product of each consecutive pair of elements is distinct, which settles a problem of Evans. In this talk, we will sketch how Evans' problem can be addressed using the randomised version of the Hall-Paige conjecture.

Friday 14:15, George Fox Lecture Theatre 2

Pattern avoiding binary trees

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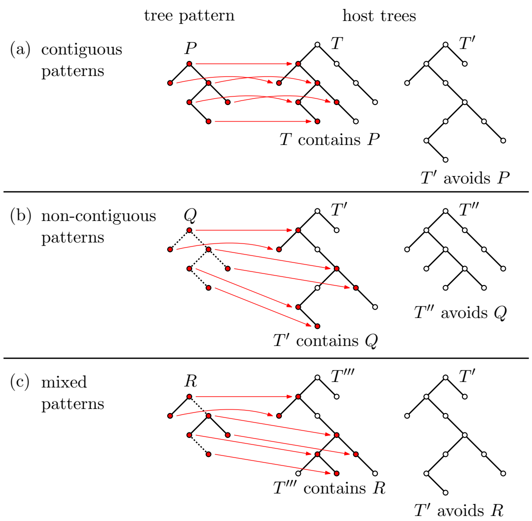
(This talk is based on joint work with Torsten Mütze.)

Pattern-avoidance is a fundamental topic in combinatorics, and in this work we consider pattern-avoidance in Catalan structures, specifically, in binary trees. The study of pattern-avoidance in binary trees was initiated by Rowland [3], who considered contiguous tree patterns, i.e., in a pattern match, the tree pattern appears as an induced subtree of the host tree; see Figure (a). Dairyko, Pudwell, Tyner and Wynn [1] considered non-contiguous tree patterns, i.e., in a pattern match, the tree pattern appears as a minor of the host tree; see Figure (b).



We generalize the two aforementioned types of tree patterns, by considering an arbitrary mix of both types, i.e., each individual edge of the tree pattern can be considered either contiguous or non-contiguous, independently of the other edges; see Figure (c).

Our first result is a bijection between the set of binary trees with n nodes that avoid any given set of such generalized tree patterns, and a set of pattern-avoiding permutations of length n. This uses mesh patterns introduced by Brändén and Claesson [4] and generalizes the earlier bijection of Pudwell, Scholten, Schrock and Serrato [2] for non-contiguous tree patterns.

Our main contribution is to apply this bijection to provide exhaustive generation algorithms for a large variety of pattern-avoiding binary trees, based on our permutation language framework [5].

[1] Dairyko, M. and Pudwell, L. and Tyner, S. and Wynn, C. Non-contiguous pattern avoidance in binary trees. *Electron. J. Combin.*, 19(3), 2012.

[2] Pudwell, L. and Scholten, C. and Schrock, T. and Serrato, A. Noncontiguous Pattern Containment in Binary Trees. *Int. Schol. Res. Not.*, Paper 316535, 9 pp, 2014.

[3] Rowland, E. S. Pattern avoidance in binary trees. *J. Combin. Theory Ser. A.*, 117(6), 2010.

[4] Brändén, P. and Claesson, A. Mesh patterns and the expansion of permutation statistics as sums of permutation patterns. *Electron. J. Combin.*, 18(2), 2011.

[5] Hartung, E. and Hoang, H. P. and Mütze, T. and Williams, A. Combinatorial generation via permutation languages. *Trans. Amer. Math. Soc.*, 375(4):2255–2291, 2022.

Friday 11:25, George Fox Lecture Theatre 3

Monochromatic arithmetic progressions in binary words associated with pattern sequences

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Let $e_v(n)$ denote the number of occurrences of a pattern v in the binary expansion of $n \in \mathbb{N}$. In the talk we consider monochromatic arithmetic progressions in the class of words $(e_v(n) \bmod 2)_{n\geq 0}$ over $\{0,1\}$, which includes the Thue–Morse word $\mathbf{t}$ (for $v = 1$) and a variant of the Rudin–Shapiro word $\mathbf{r}$ (for $v = 11$). So far, the problem of exhibiting long progressions and finding an upper bound on their length has mostly been studied for $\mathbf{t}$ and certain generalizations [1, 2, 3]. The main goal of the talk is to show analogous results for $\mathbf{r}$ and some weaker results for a general pattern v. In particular, we prove that a monochromatic arithmetic progression of difference $d \geq 3$ starting at 0 in $\mathbf{r}$ has length at most $(d+3)/2$, with equality infinitely often. We also compute the maximal length of monochromatic progressions of differences $2^k - 1$ and $2^k + 1$.

[1] I. Aedo, U. Grimm, Y. Nagai, P. Staynova, *On long arithmetic progressions in binary Morse-like words*, preprint, https://arxiv.org/abs/2101.02056 (2021), 23 pp.

[2] J. F. Morgenbesser, J. Shallit, T. Stoll, *Thue–Morse at multiples of an integer*, J. Number Theory **131** (2011), no. 8, 1498–1512.

[3] O. G. Parshina, *On arithmetic index in the generalized Thue–Morse word*, in: S. Brlek, F. Dolce, C. Reutenauer, É. Vandomme (eds.), Combinatorics on Words, Springer, Cham, 2017, 121–131

Friday 11:50, George Fox Lecture Theatre 3

Caps up to dimension 7

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A cap of size s in dimension n is given by a collection of s points, no three of which are collinear, in n-dimensional affine space over the field of three elements. The cap set problem asks for the largest possible size of a cap in each dimension. The problem is solved for dimensions up to and including 6, but is open for dimensions 7 and higher.

We use the results of computer searches to classify large caps in dimensions 5 and 6, and to prove that in dimension 7, the size of every cap is at most 288.

This talk is based on two upcoming papers by the author (Thackeray 2022a-b). The research was supported by the UP Post-Doctoral Fellowship Programme administered by the University of Pretoria (grant number A0X 816).

Thackeray, H. (M.) R. 2022a. The cap set problem: 41-cap 5-flats. In preparation.

Thackeray, H. (M.) R. 2022b. The cap set problem: Up to dimension 7. In preparation.

Turán numbers of sunflowers

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(This talk is based on joint work with Matija Bucić and Benny Sudakov.)

A collection of distinct sets is called a *sunflower* if the intersection of any pair of sets equals the common intersection of all the sets. Sunflowers are fundamental objects in extremal set theory with relations and applications to many other areas of mathematics as well as to theoretical computer science. A central problem in the area due to Erdős and Rado from 1960 asks for the minimum number of sets of size r needed to guarantee the existence of a sunflower of a given size. Despite a lot of recent attention including a polymath project and some amazing breakthroughs, even the asymptotic answer remains unknown.

We study a related problem first posed by Duke and Erdős in 1977 which requires that in addition the intersection size of the desired sunflower be fixed. This question is perhaps even more natural from a graph theoretic perspective since it asks for the Turán number of a hypergraph made by the sunflower consisting of k edges, each of size r and with common intersection of size t. For a fixed size of the sunflower k, the order of magnitude of the answer has been determined by Frankl and Füredi. In the 1980's, with certain applications in mind, Chung, Erdős and Graham considered what happens if one allows k, the size of the desired sunflower, to grow with the size of the ground set. In the three uniform case, $r = 3$, the correct dependence on the size of the sunflower has been determined by Duke and Erdős and independently by Frankl and in the four uniform case by Bucić, Draganić, Sudakov and Tran. We resolve this problem for any uniformity, by determining up to a constant factor the n-vertex Turán number of a sunflower of arbitrary uniformity r, common intersection size t and with the size of the sunflower k allowed to grow with n.

1-INDEPENDENT PERCOLATION IN $\mathbb{Z}^2 \times K_n$

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(This talk is based on joint work with Victor Falgas-Ravry.)

A random graph model on a host graph H is said to be 1-*independent* if for every pair of vertex-disjoint subsets A, B of $E(H)$, the state of edges (absent or present) in A is independent of the state of edges in B. For an infinite connected graph H, the 1-*independent critical percolation probability* $p_{1,c}(H)$ is the infimum of the $p \in [0,1]$ such that every 1-independent random graph model on H in which each edge is present with probability at least p almost surely contains an infinite connected component.

Balister and Bollobás observed in 2012 that $p_{1,c}(\mathbb{Z}^d)$ is nonincreasing and tends to a limit in $[\frac{1}{2}, 1]$ as $d \to \infty$. They asked for the value of this limit. We make progress towards this question by showing that

$$\lim_{n\to\infty} p_{1,c}(\mathbb{Z}^2 \times K_n) = 4 - 2\sqrt{3} = 0.5358\ldots\ .$$

In fact, we show that the equality above remains true if the sequence of complete graphs K_n is replaced by a sequence of weakly pseudorandom graphs on n vertices with average degree $\omega(\log n)$. We conjecture that the equality also remains true if K_n is replaced instead by the n-dimensional hypercube Q_n. This latter conjecture would imply the answer to Balister and Bollobás's question is $4 - 2\sqrt{3}$.

Friday 11:50, George Fox Lecture Theatre 5

Exchange distance of basis pairs in split matroids

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(This talk is based on joint work with Kristóf Bérczi.)

The basis exchange axiom has been a driving force in the development of matroid theory. However, the axiom gives only a local characterization of the relation of bases, which is a major stumbling block to further progress, and providing a global understanding of the structure of matroid bases is a fundamental goal in matroid optimization.

While studying the structure of symmetric exchanges, Gabow proposed the problem that any pair of bases admits a sequence of symmetric exchanges. A different extension of the exchange axiom was proposed by White, who investigated the equivalence of compatible basis sequences. Farber studied the structure of basis pairs, and conjectured that the basis pair graph of any matroid is connected. These conjectures suggest that the family of bases of a matroid possesses much stronger structural properties than we are aware of.

In the present talk, we study the distance of basis pairs of a matroid in terms of symmetric exchanges. In particular, we give an upper bound on the minimum number of exchanges needed to transform a basis pair into another for split matroids, a class that was motivated by the study of matroid polytopes from a tropical geometry point of view. As a corollary, we verify the above mentioned long-standing conjectures for this large class. Being a subclass of split matroids, our result settles the conjectures for paving matroids as well.

Friday 13:50, George Fox Lecture Theatre 5

Symmetry and the design of self-stressed structures

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(This talk is based on joint work with Cameron Millar (SOM), Arek Mazurek (Mazurek Consulting) and William Baker (SOM).)

In 2000 Fowler and Guest established a symmetry-extended Maxwell rule for the rigidity of (bar-joint) frameworks. This rule can often reveal 'hidden' infinitesimal motions and states of self-stress in symmetric frameworks that cannot be detected with Maxwell's original rule from 1864. In this talk we show how this rule can be used to derive an efficient new method for constructing symmetric frameworks with a large number of 'fully-symmetric' or 'anti-symmetric' states of self-stress. Maximizing the number of independent states of self-stress of a planar framework, as well as understanding their symmetry properties, has important practical applications, for example in the design and construction of gridshells. We show the usefulness of our method by applying it to some practical examples.

The geometry of random graphs with a Markov flavour

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Random graphs are at the intersection of probability and graph theory: it is the study of the stochastic process by which graphs form and evolve. In 1959, Erdős and Rényi defined the foundational model of random graphs on n vertices. Subsequently, Frank and Strauss (1986) added a Markov twist to this story by describing a topological structure on random graphs that encodes dependencies between local pairs of vertices. The general model that describes this framework is called the exponential random graph model (ERGM). It is used in social network analysis and appears in statistical physics as in the ferromagnetic Ising model. We characterize the parameters that determine when an ERGM has desirable properties using a well-developed dictionary between probability distributions and their corresponding generating polynomials.

Index