# Input Uncertainty Quantification in Stochastic Simulation

Daniel Morton

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Quantifying Input Uncertainty

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#### Content

- A brief introduction.
- Motivation who's interested in input uncertainty?
- Different solutions.
- Illustrate findings.

Stochastic simulation is used for many real-world applications. There are 2 sources of variance that arise from stochastic simulation:

- Stochastic uncertainty.
- Input uncertainty.

The sum of these 2 sources contribute to the total variance of the simulation.

## What is input uncertainty?

- Input models are used to drive simulations. We use real-world data to estimate these input models.
- Input uncertainty describes the error from having estimated these input models from finite data.
- Ultimately, incorrect input models will lead to incorrect results.

#### Motivation

- Input uncertainty is usually ignored all together!
- Where data sets are small, input uncertainty can be much greater than stochastic uncertainty.
- Only stochastic uncertainty is usually reported. This will give a false sense of confidence when analysing the outputs from the simulation.

• Bootstrap method.

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- Bootstrap method.
- Subsampling extension to Bootstrap.

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- Bootstrap method.
- Subsampling extension to Bootstrap.
- Taylor-series approximation.

$$\sigma_I^2 \approx g(\hat{\theta})^T \operatorname{Var}(\hat{\theta}) g(\hat{\theta})$$

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- Bootstrap method.
- Subsampling extension to Bootstrap.
- Taylor-series approximation.

$$\sigma_I^2 \approx g(\hat{\theta})^T \operatorname{Var}(\hat{\theta}) g(\hat{\theta})$$

Assumption - we know the distribution family from which we are estimating the parameters.

### Taylor-series approximation

- Gives a more accurate estimation than Bootstrapping and Subsampling.
- Much faster to run! Only requires 1 simulation (Bootstrapping and Subsampling require BR simulations.)
- Shows contribution of individual parameters.
- Easy to extend to more realistic queuing models.

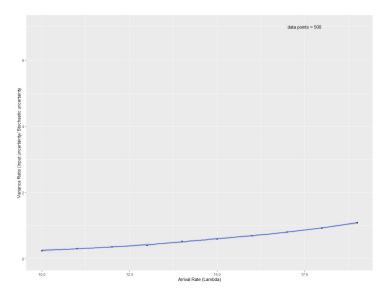
### Queuing model

$$\xrightarrow{\lambda} (c_1, s_1, \mu_1) \xrightarrow{p_i} (c_2, s_2, \mu_2) \xrightarrow{q_i} (c_3, s_3, \mu_3)$$

$$\downarrow 1 - p_i \qquad \qquad \downarrow 1 - q_i$$

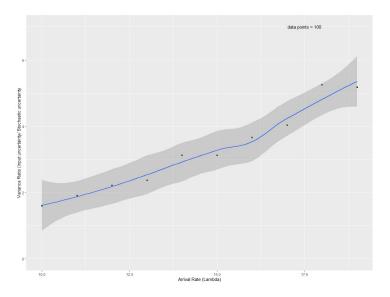
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## Findings



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## Findings



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## Thanks for listening!

• Any questions?

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