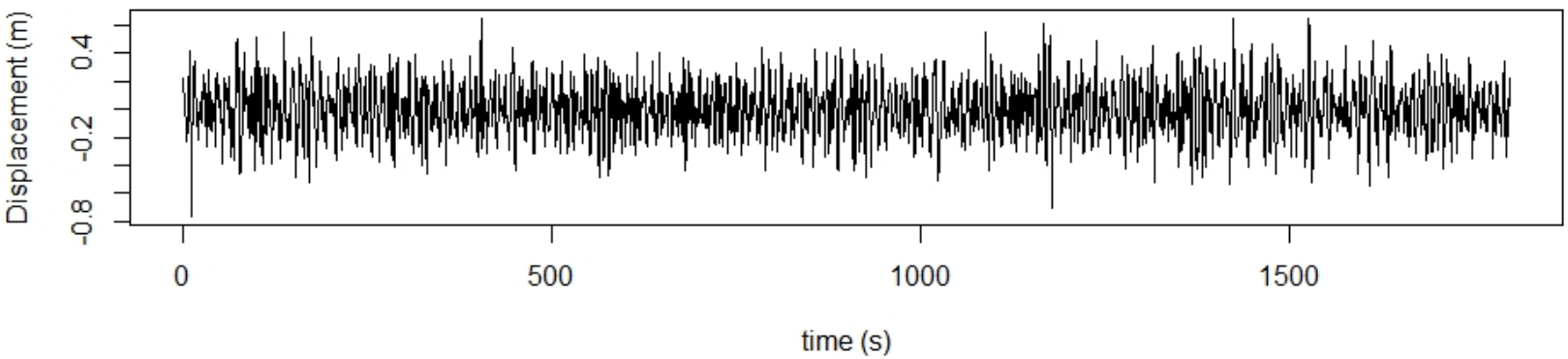
Modelling Waves in the Ocean

Jack McGinn
Supervisor: Jake Grainger

September 2020

Time Series

- Wind generated waves treated as stochastic process
- Sampling displacement over time of some point in the ocean is denoted by stochastic variable $\mathbf{X}_\Delta = [X_{t\Delta}]_{t\in\mathbb{Z}}$ forming discrete time series



Spectral Density

- Continuous Spectral Density

$$f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} c(\tau) exp(-i\omega\lambda) d\tau$$

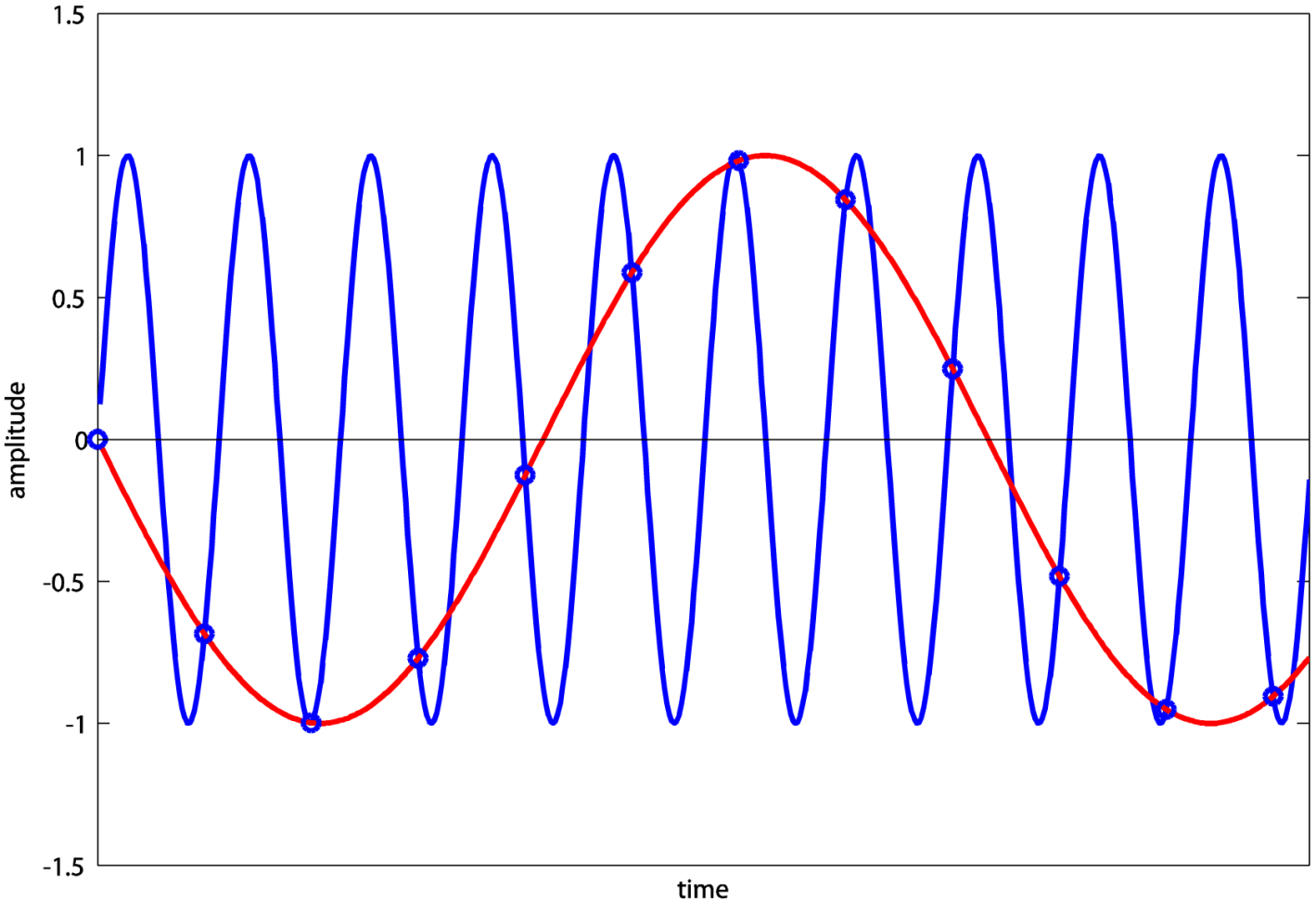
- Discrete Spectral Density

$$f_{\Delta}(\omega) = \frac{\Delta}{2\pi} \sum_{\tau=-\infty}^{\infty} c(\tau\Delta) exp(-i\omega\tau\Delta)$$

- Aliasing

$$f_{\Delta}(\omega) = \sum_{k=-\infty}^{\infty} f(\omega + \frac{2\pi k}{\Delta})$$

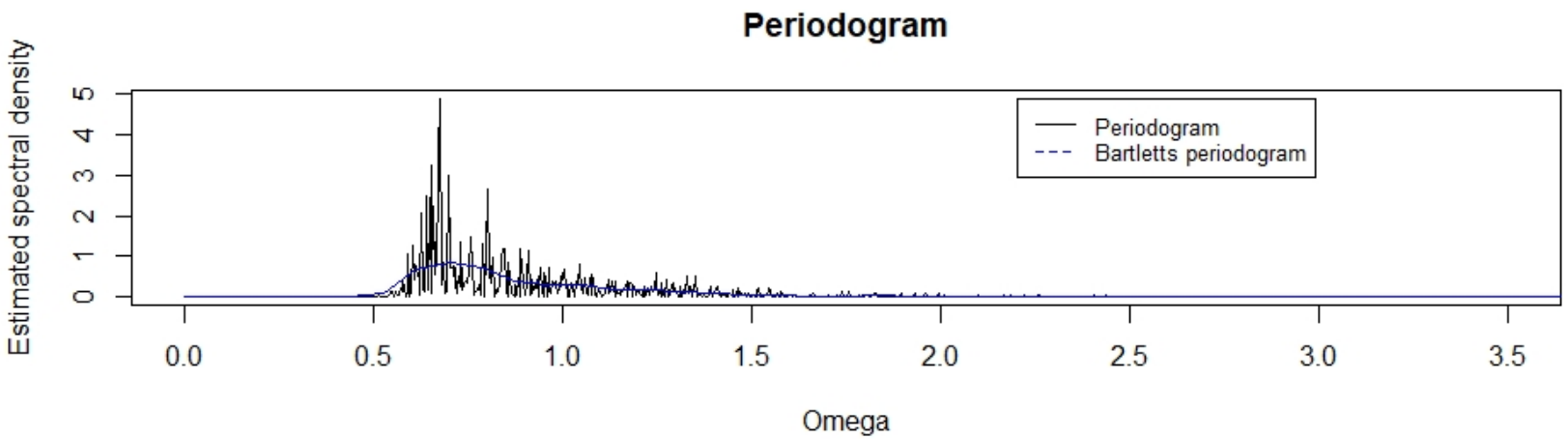
Aliasing



Periodogram

Periodogram estimate for spectral density from wave displacement time series

$$I(\omega) = \frac{\Delta}{2\pi N} \left| \sum_{t=0}^{N-1} X_{t\Delta} exp(-it\Delta\omega) \right|^2$$

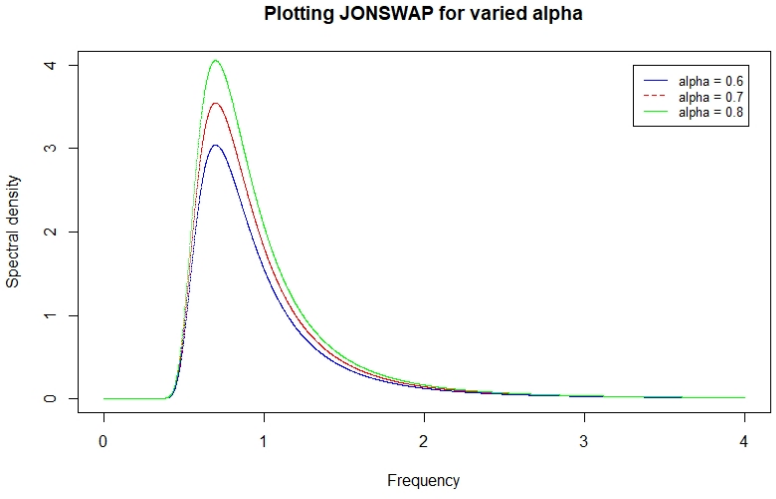


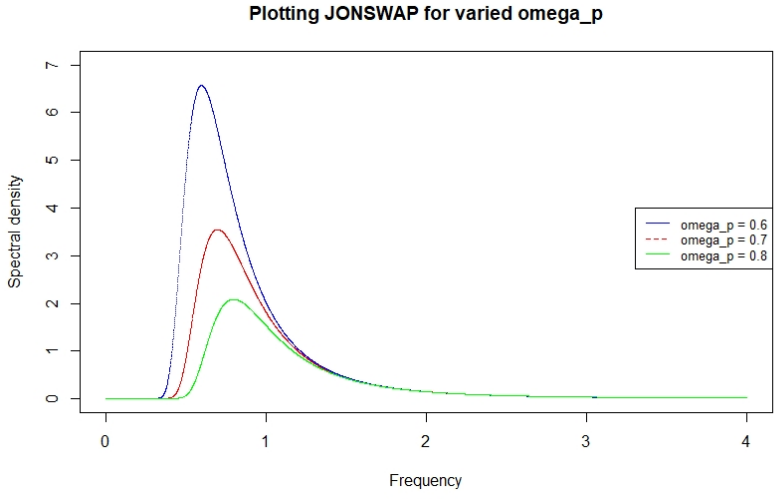
JONSWAP

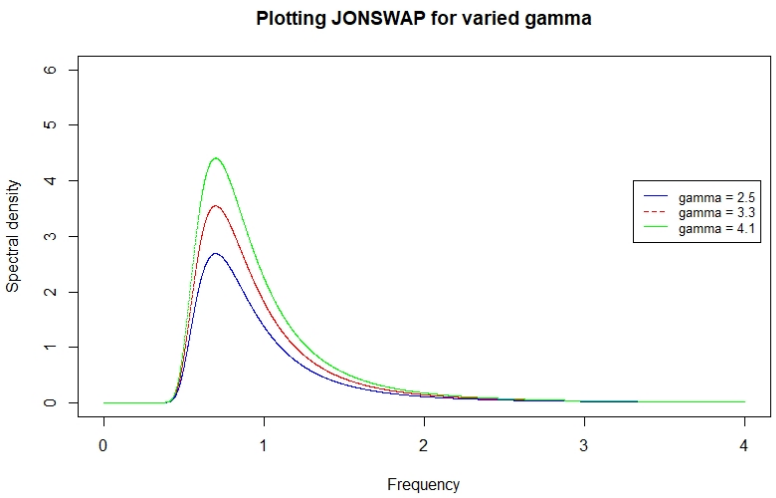
JONSWAP model for spectral density of wind generated waves in the ocean

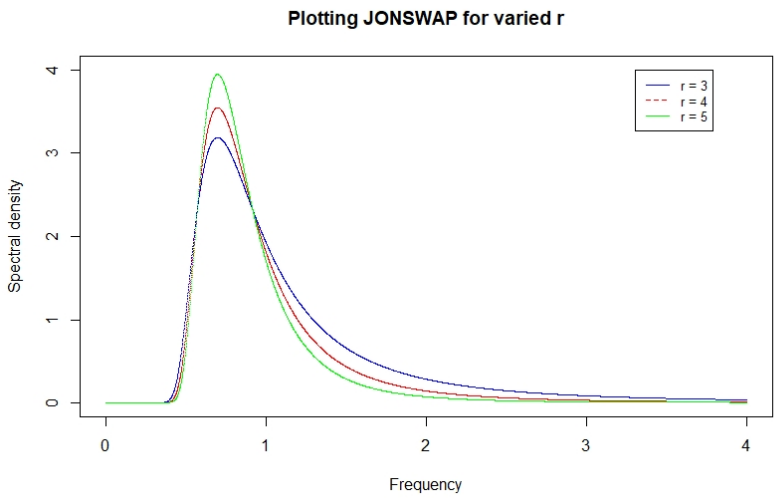
$$S_G(\omega|\theta) = \alpha\omega^{-r} exp\left(\frac{-r}{s}\left(\frac{\omega}{\omega_p}\right)^{-s}\right)\gamma^{\delta(\omega|\theta)}$$

JONSWAP


Plotting JONSWAP for varied alpha
alpha = 0.6
alpha = 0.7
alpha = 0.8
Spectral density
Frequency
0
1
2
3
4


Plotting JONSWAP for varied omega_p
omega_p = 0.6
omega_p = 0.7
omega_p = 0.8
Spectral density
Frequency
0
1
2
3
4
5
6
7


Plotting JONSWAP for varied gamma
gamma = 2.5
gamma = 3.3
gamma = 4.1
Spectral density
Frequency
0
1
2
3
4
5
6


Plotting JONSWAP for varied r
r = 3
r = 4
r = 5
Spectral density
Frequency
0
1
2
3
4

Estimating parameters - Whittle Approximation

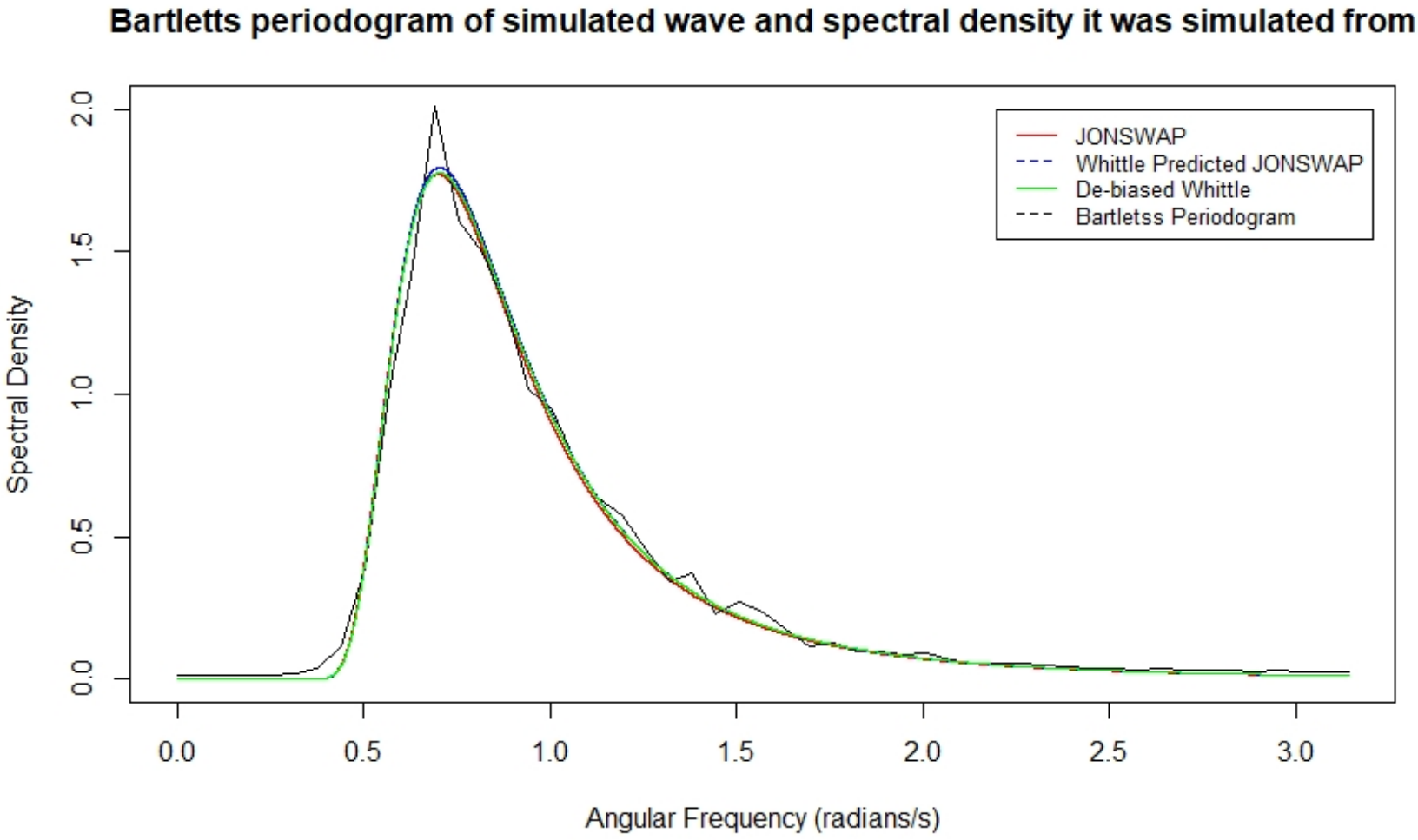
- Whittle Likelihood function

$$\ell_W(\theta|X_{\Delta,N}) = -\sum_{\omega\in\Omega} log(f(\omega|\theta)) + \frac{I(\omega)}{f(\omega|\theta)}$$

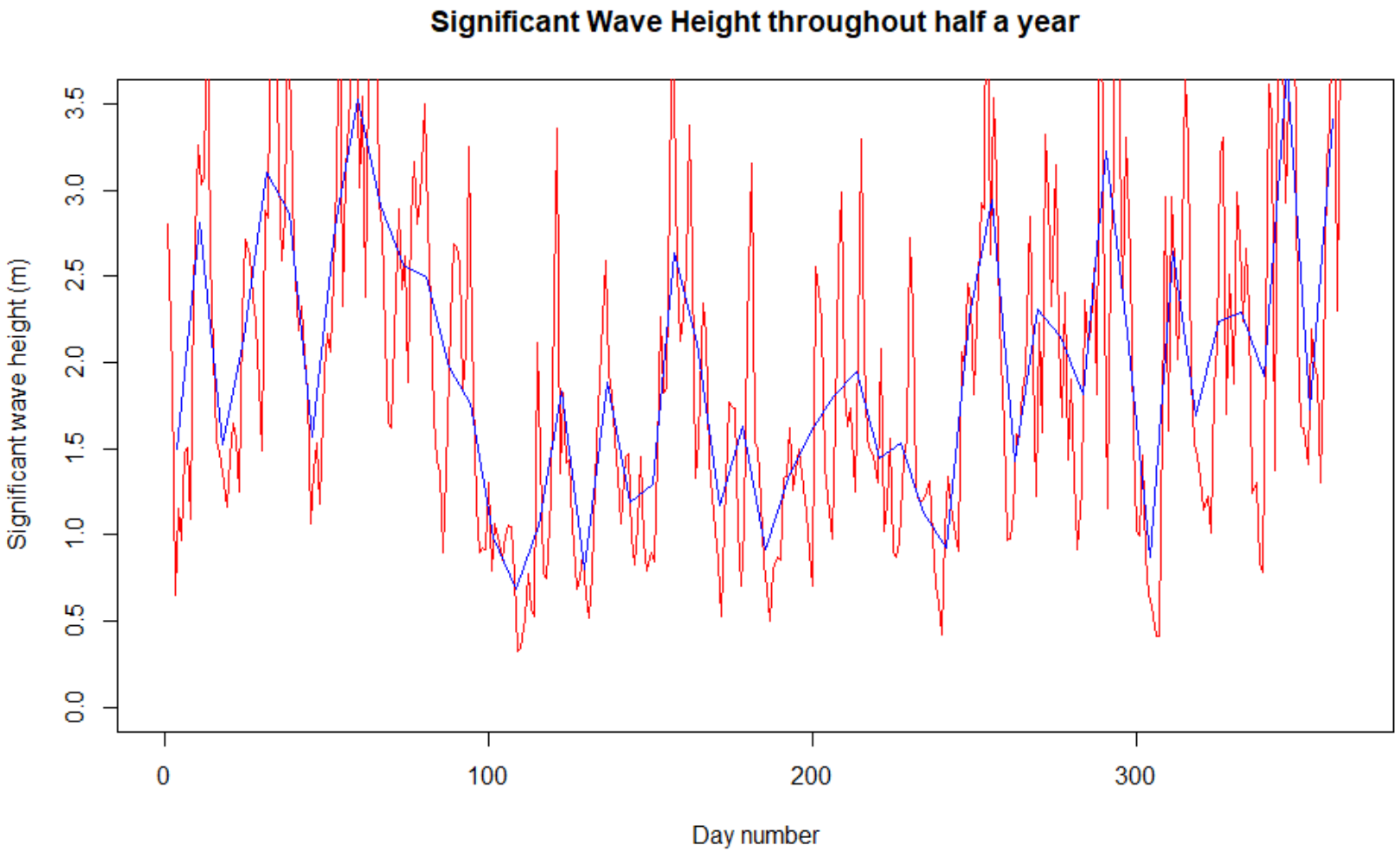
- De-biased Whittle Likelihood function, replace $f(\omega|\theta)$ in fraction with,

$$E[I(\omega)] = \frac{1}{2\pi} Re\left(2\Delta \sum_{\tau=0}^{N-1} \left(1 - \frac{\tau}{N}\right) c(\tau|\theta) exp(-i\omega\tau\Delta) - \Delta c(0|\theta)\right)$$

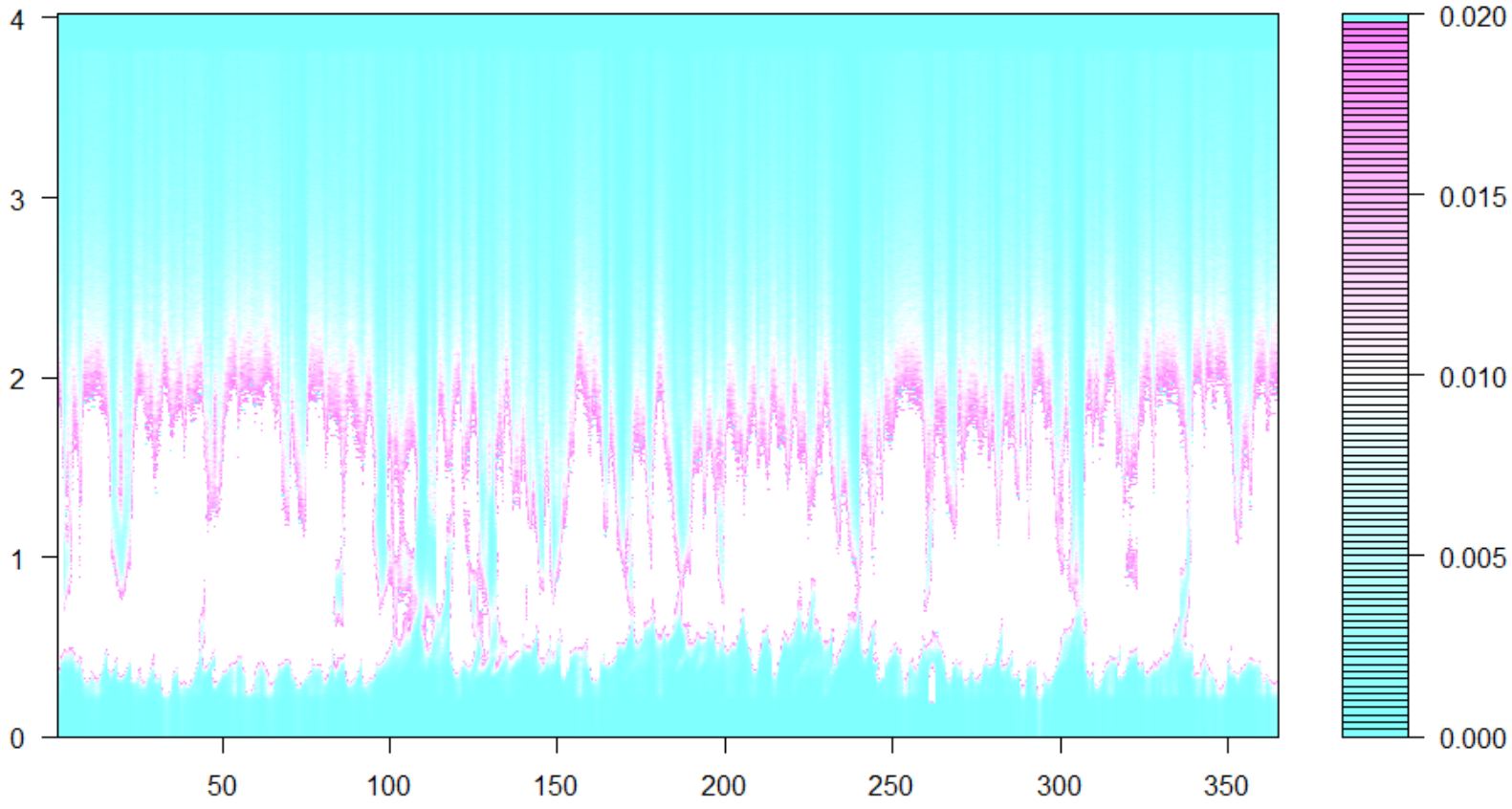
Simulated Data



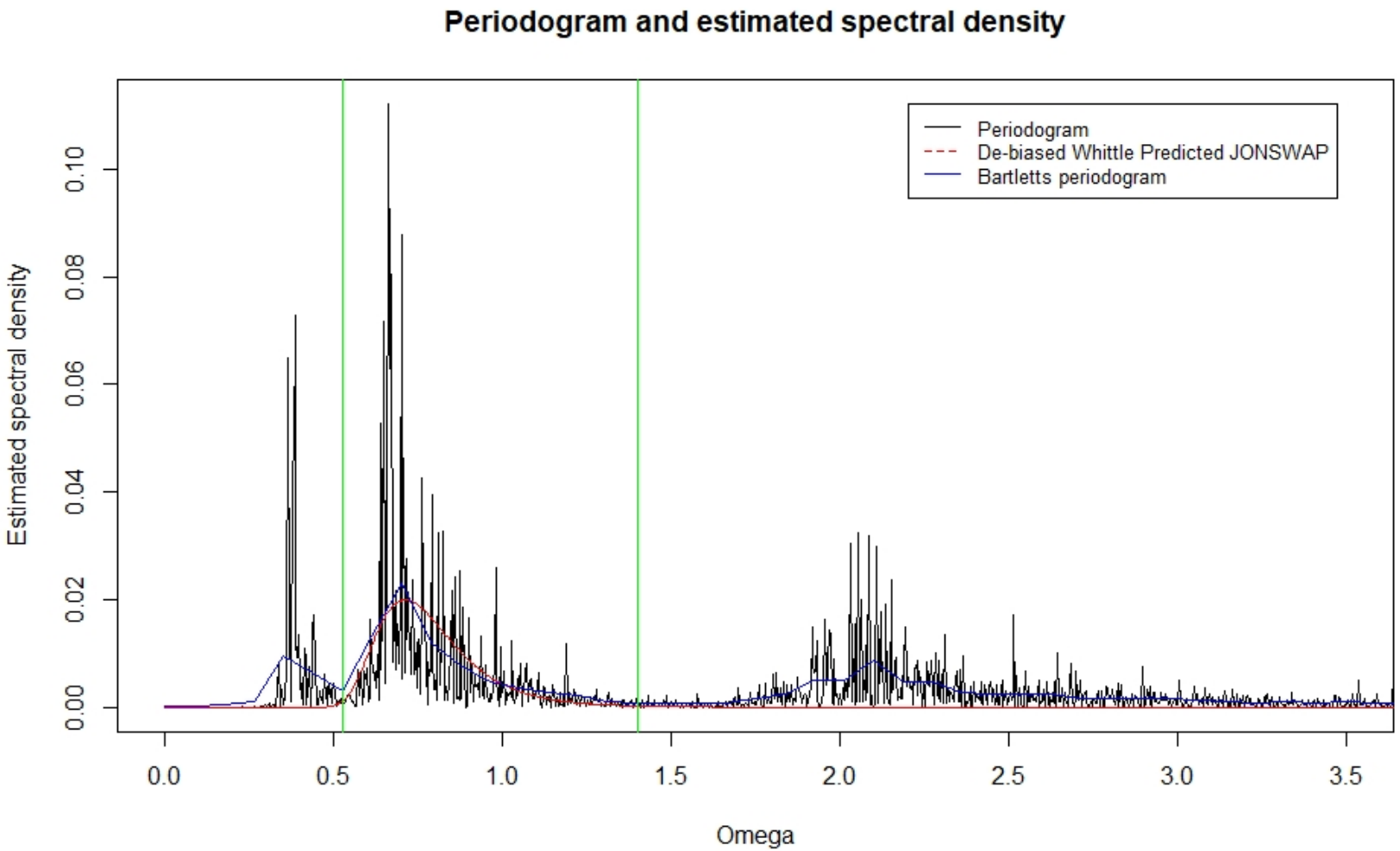
Real Data


Significant Wave Height throughout half a year
Significant wave height (m)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
100
200
300
Day number

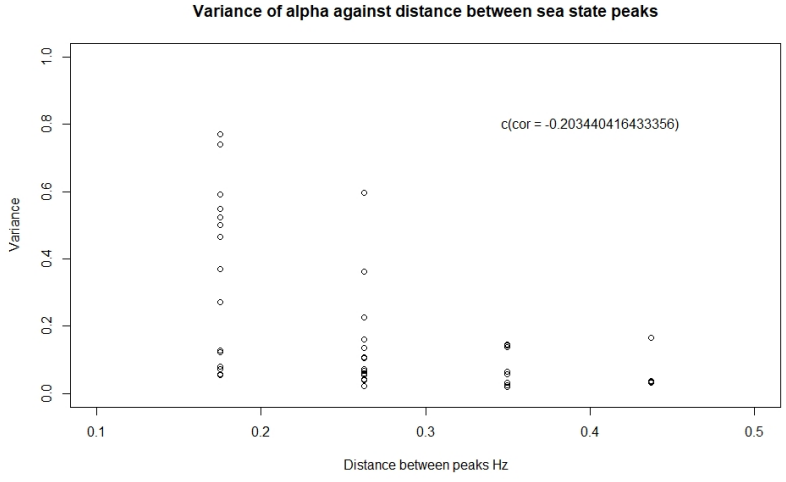
Real Data continued

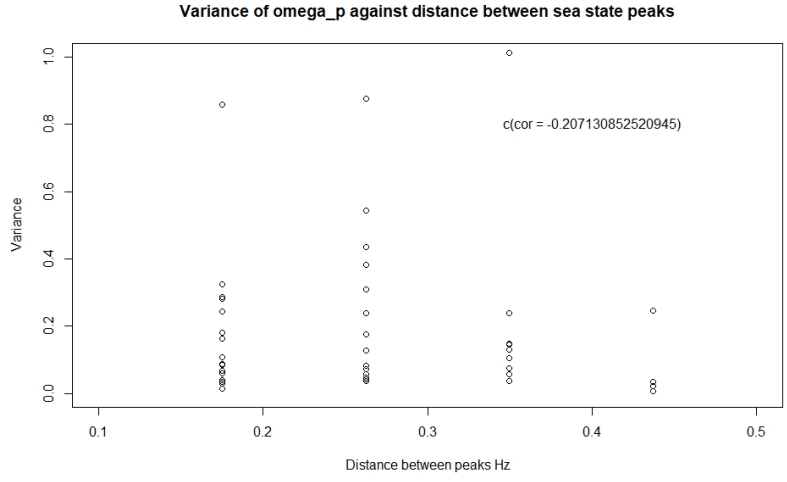


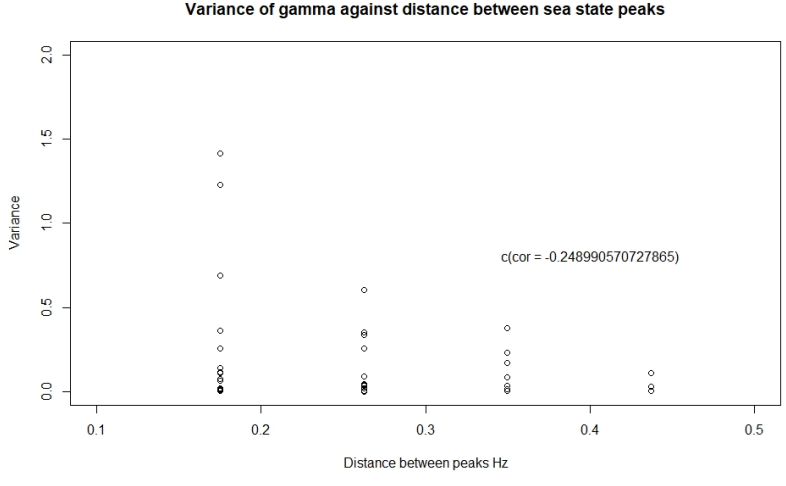
Swell

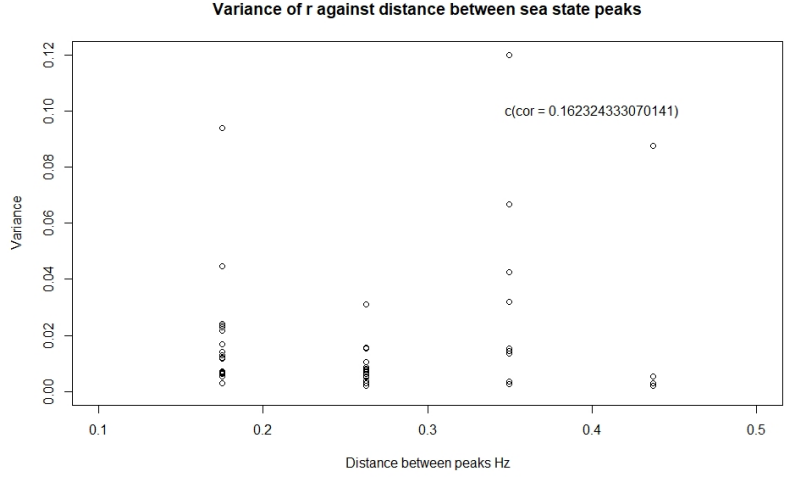

Periodogram and estimated spectral density
Periodogram
De-biased Whittle Predicted JONSWAP
Bartletts periodogram
Estimated spectral density
0.00
0.02
0.04
0.06
0.08
0.10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Omega

Robustness of Removing swell

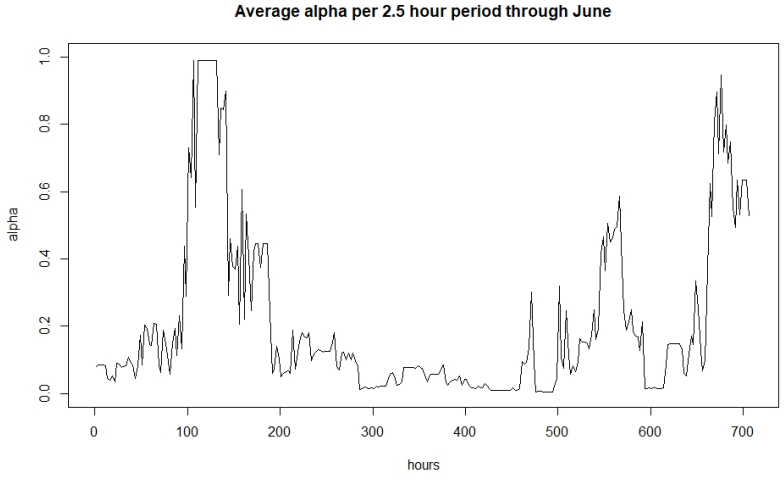


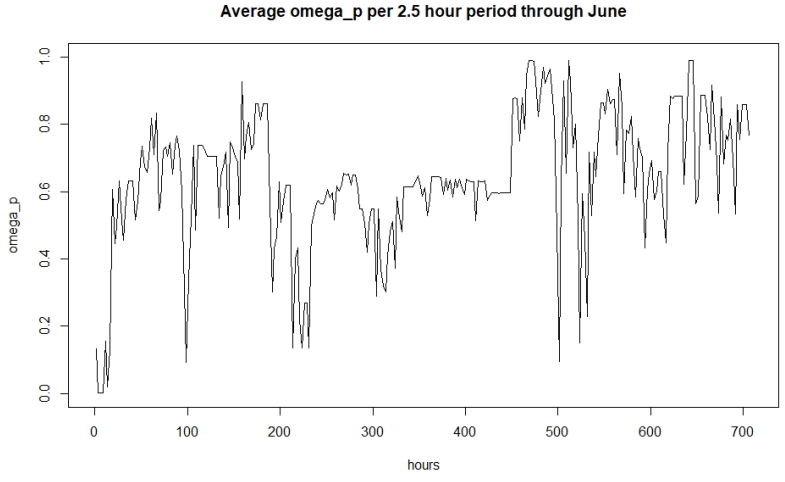


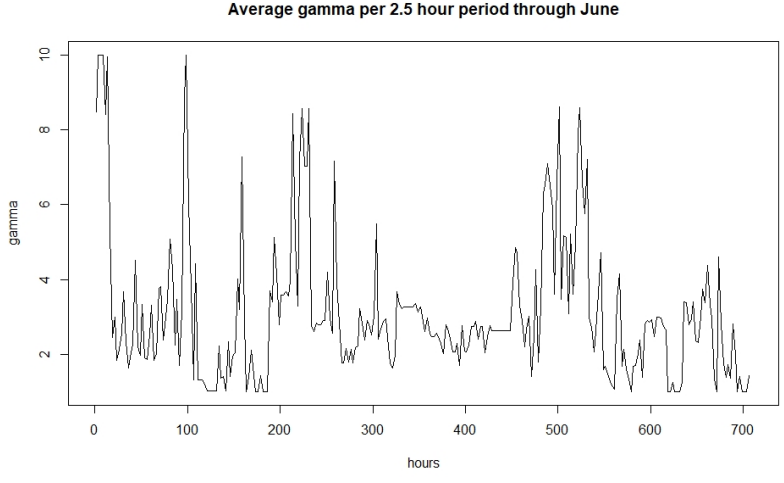


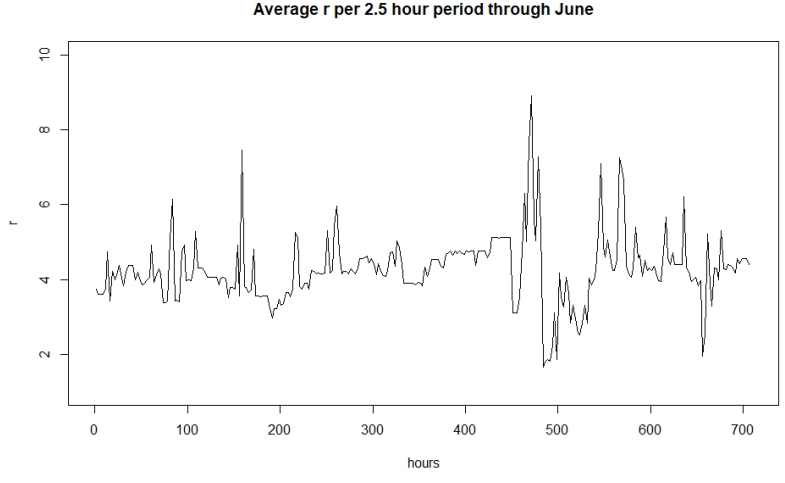


Real Parameters









Further Work

- Improve method of testing robustness
- Improve method for removing swell