Anomaly Detection Using FDA

With Applications to Sea Surface Temperature Data

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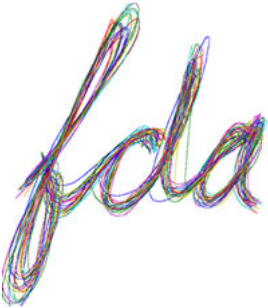
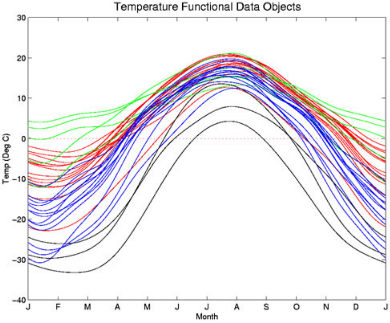
Overview

1. Introduction to FDA
2. The Dataset
3. Anomaly Detection

Functional Data Analysis

- Functional Data Analysis (FDA) involves the analysis of information on curves or functions.
- No assumptions (such as stationarity, low dimensionality, equally spaced observations ect.) have to be made about the functions or the data.





Functional Data Analysis - Basis Functions

- We use *basis function expansions* to model functions:

  $$x(t) = a_1\phi_1(t) + a_2\phi_2(t) + ... + a_k\phi_k(t) = \sum_{i=1}^{k} a_i\phi_i$$

  $\phi_i(t)$ is the i-th basis function
  a_i are constant coefficients

- Basis functions are essentially building blocks that make up our functions.

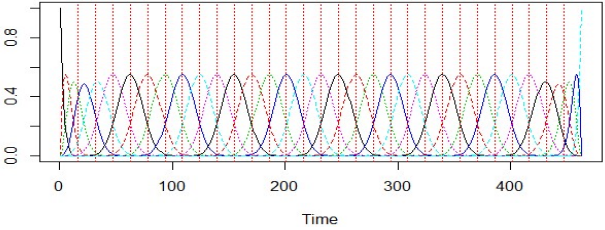
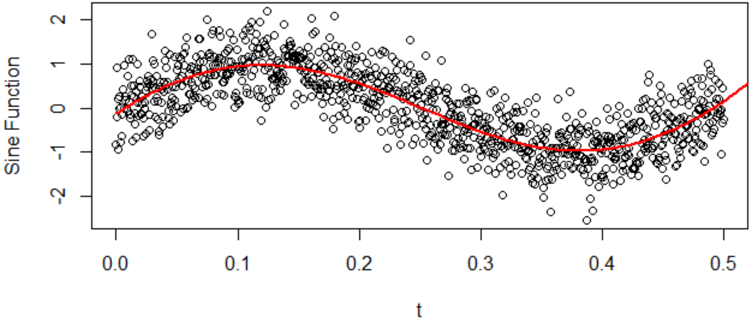


Figure: Order 6 B-spline basis function

Functional Data Analysis - Smoothing

- We want to eliminate high local variation within the data but retain a good fit.
- We minimise the Sum of Squared Error but add a *Roughness Penalty*.
- The smoothing parameter λ determines how much we smooth the data.



The Dataset

- The dataset consists of the Sea Surface Temperature recorded every month (for almost 40 years) at 2000 different locations in the North Atlantic Ocean.

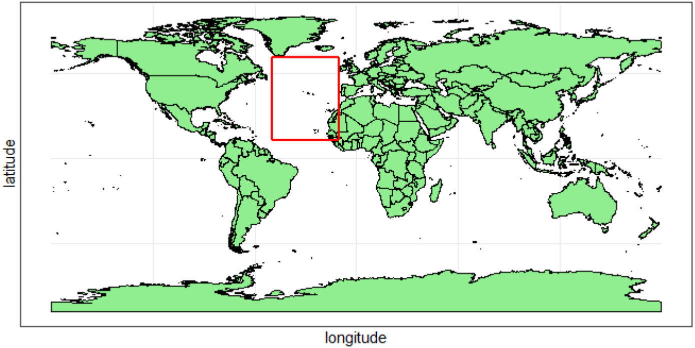
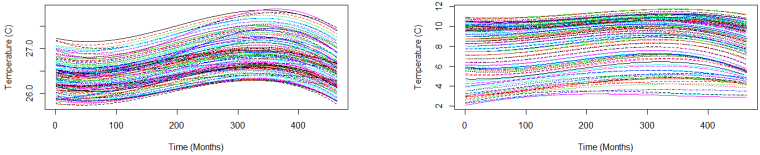


Figure: Dataset Location

The Dataset - FDA

- The discrete dataset is converted into functional form, with each curve representing one location.
- The data is smoothed using a smoothing spline with the smoothing parameter (λ) determined by GCV.



(a) Most Southerly 100 locations

(b) Most Northerly 100 locations

K-means Clustering

- I wanted to split the large dataset into smaller groups and K-means clustering seemed a sensible way to do this.
- The functions were clustered in this way to group locations with similar climates.
- The data was split into 15 clusters as this minimised AIC.

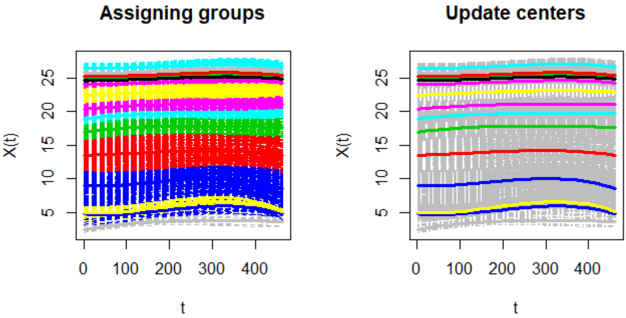
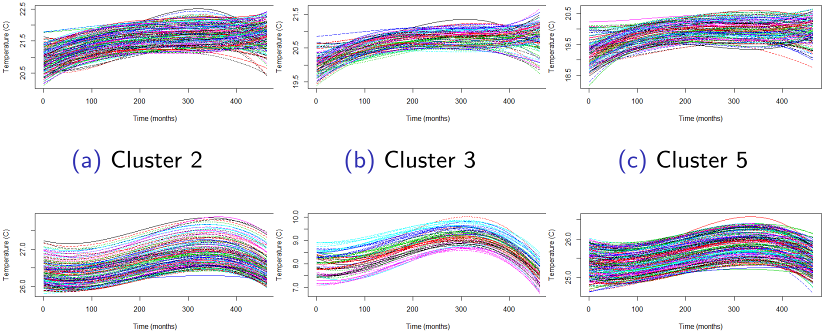


Figure: K-means Clusters

K-means Clustering - Continued

- The set is split into 15 clusters and I have used anomaly detection methods for functional data in order to detect outlying curves within each cluster.



(a) Cluster 2

(b) Cluster 3

(c) Cluster 5

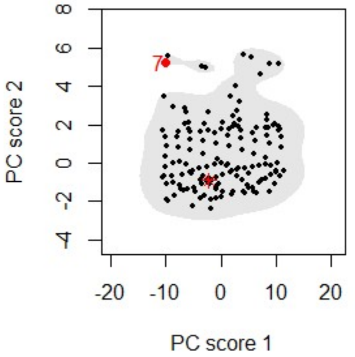
(d) Cluster 6

(e) Cluster 10

(f) Cluster 14

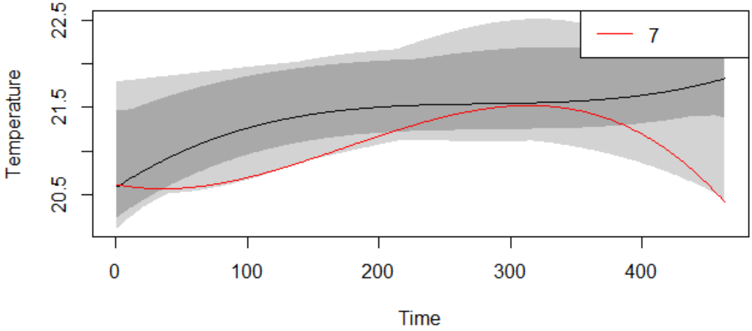
Anomaly Detection

- A *principle component analysis* is used to decompose functional data into the first two principle components and their principle component scores.
- The first two principle component scores are used to make the anomaly detection multivariate rather than functional.



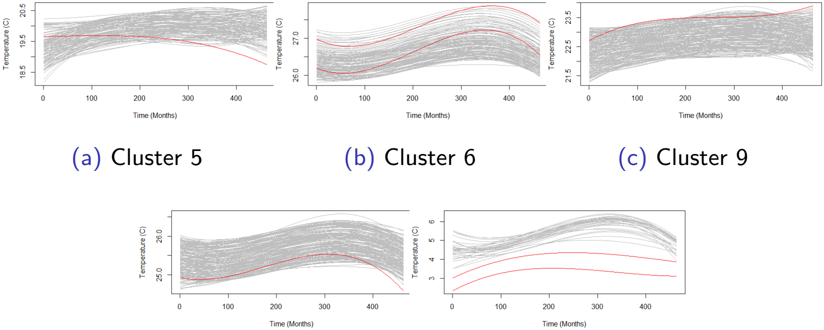
Anomaly Detection - Continued

- Outliers in functional data can be identified as outliers in the *bivariate score space*.
- The score space here is the *highest density region*.
- Any curve outside this region can be considered anomalous.



Anomaly Detection - Sea Surface Temperature Data

- I used this method to detect outliers within the Sea Surface Temperature dataset.
- I focused on curves outside of the 99.9% highest density region; within each cluster.



(a) Cluster 5

(b) Cluster 6

(c) Cluster 9

(d) Cluster 14

(e) Cluster 15

Anomaly Detection - Locations

- The anomalous locations are almost exclusively on the coast of Africa and Greenland.

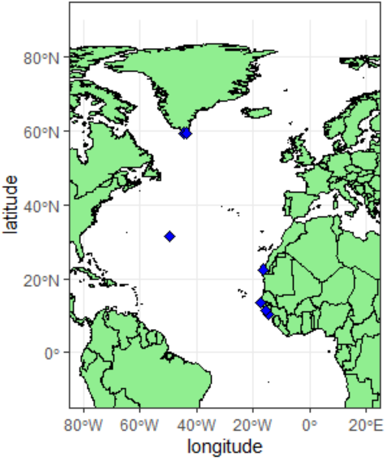


Figure: Locations of Outliers

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